

CHRISTIANITY AND MATHEMATICS

An Analysis of Differing Approaches to their Interrelationship

by Calvin Jongsma

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INTRODUCTION

The last two decades have witnessed the growth of a rather remarkable phenomenon: Christian mathematicians discussing the integration of their Christian faith with their mathematical work. This cannot be a totally new turn of events - I'm sure Christian mathematicians have always reflected on this issue to some extent - but the increased number of articles and talks dealing with this matter in the past two decades makes me suspect that there is a new spirit or current of thought at work, particularly among Evangelical, Protestant Christians. Actually, it would be more accurate to put this observation in the plural, for the trend toward relating Christianity and mathematics is in no way a movement with a single direction, as we will see.

Many people will be surprised to learn that there is any resource material on this topic at all, regardless of perspective; the rest will probably be surprised, as I was, to discover just how much has been written. The booklet BIBLIOGRAPHY OF CHRISTIANITY AND MATHEMATICS: 1910 - 1983, which Gene Chase of Messiah College and I recently compiled and annotated, contains some 300 items in this genre. The surprise such a figure evokes from people is a good indication of the degree of separation which has existed among Christian scholars and students of mathematics. Less than ten years ago (1977), a first conference was called to touch upon this issue in a concerted manner. Under the leadership of Dr. Robert Brabenec of the Wheaton College mathematics department, about 120 conferees gathered to discuss the possibility and contours of a Christian perspective on the foundations of mathematics. Prior to the conference Bob made available nine articles for people to study; this list exhausted all the articles on the topic of Christianity and mathematics known to the organizers at the time. Since then, English speaking Christian mathematicians have been put in closer contact with one another through a series of five biennial Wheaton Conferences focusing on the general theme of Christianity and mathematics. Our BIBLIOGRAPHY appeared in 1983, giving those who wished to study the literature ready access to articles and papers treating a wide range of topics from a variety of perspectives.

Communal awareness of the issues involved in relating Christianity and mathematics has thus increased since 1977, but one can hardly say that a consensus of opinion on the various topics has been achieved. Coming together to hear talks and share ideas with one another and taking time to read and write articles on this matter has greatly facilitated interaction between Christian mathematicians, but a close analysis of the literature reveals a great diversity, even divergence, in approach, goals, and content.

There are marked differences in quality as well, for many items that we uncovered were never intended as serious, academic publications. The topic seems particularly popular among people involved on various levels with mathematics education in Christian schools. Many articles (including this one) originated as talks or papers written for some particular educational purpose. Further, some Christian colleges require tenure-stream faculty to write position papers on the relation between faith and learning in their particular area; this has given rise to other kinds of papers. Some of these talks and papers are insightful and fairly well written, but the fact remains that they are usually not done with the same degree of professional care and attention as those which are submitted to a refereed journal. Relatively few works are written with the requisite knowledge or expertise to command respect (if not agreement) from secular mathematicians, historians of mathematics, and philosophers of mathematics.

In clarifying the differences in approach which exist in the literature, my intent is to help us all become more aware of some of the religious and philosophical underpinnings of the various viewpoints on mathematics. Being more aware of one another's outlooks on these levels, I think we will be able to communicate on a deeper level than we are presently doing. As we attempt to formulate our positions more exactly over against other ones, the quality of our writing and discussion will undoubtedly improve as well.

Some may wonder whether a dialogue over the religious differences which exist between Christian mathematicians won't hinder the development of a Christian philosophy of mathematics, thinking that such a discussion will get bogged down in unrelated theological issues. I strongly sympathize with this attitude. I do not believe that all doctrinal differences need to be resolved before Christian mathematicians can work together to shape a common perspective on their field. But what cannot be ignored are fundamental differences in world-view, differences in approach to the perennial problem of Christ and culture, faith and reason, the Word of God and science, and so on. Since the topic of the relation between Christianity and mathematics is part and parcel of this much larger issue, it would be good for all of us Christian mathematicians to be conscious of and open about our general approach, even as we discuss more specific ideas with one another. Unless differences in orientation are recognized for what they are, we will end up talking past one another. This will only add unnecessary confusion to our dialogue and so impede further progress in developing a unified Christian philosophy of mathematics.

My aim in this paper, then, is to give a beginning analysis of a number of different approaches to the relation between Christianity and mathematics. Though I do not agree with some of the connections which others have postulated, I will attempt to describe the main features of each approach as fairly

as I can. I will sketch my own outlook last and in more detail than the others, though it, too, will be far from complete. Finally, I will briefly draw out some basic implications of my standpoint for the philosophy of mathematics and mathematics education.

#### THE "TWO REALMS WITH NO INTERCONNECTIONS" APPROACH

One alternative regarding the relationship between Christianity and mathematics is the extreme position that there is no relationship whatsoever. Mathematical theory and practice have no intrinsic or extrinsic connections with religion, Christian or otherwise. Christianity has nothing to say about mathematics, and mathematical theories have no implications for religious beliefs. Religion and mathematics form two disjoint realms with absolutely no interaction between them. The world of mathematics is completely untainted by subjective religious biases, and it in turn makes no professional demands on the religious outlook of its practitioners.

This viewpoint is probably the dominant one of most secular mathematicians. If such persons were confronted with an affirmative answer to the question, "Is there any connection between the Christian religion and mathematics?", they would most likely dismiss it as obscurantist nonsense. Christian mathematicians generally adopt a different stance on this issue, though at times they seem to lean in this direction. In their case, however, I suspect it is largely out of default. Since there has been no developed Christian mind on how mathematics and the Christian faith interrelate, one may not have a good defense against this two realm, no contact approach in a given situation.

#### THE "TWO REALMS WITH EXTERNAL CONNECTIONS" APPROACH

A position which many Christians do find attractive is the one which rejects any essential relationship between mathematics and Christianity in favor of certain external relationships. Christianity is related to mathematics not with respect to its subject matter, but only with respect to its use and communication. The Christian faith has bearing on mathematics primarily or solely insofar as people are involved in applying it; that is, Christian principles come into play on an ethical and interpersonal level. One must use mathematical results purely for salutary purposes, always seeking the well-being of one's neighbor. The misuse of statistics, the reduction of people to numbers, and the use of mathematical theories for evil or destructive ends have all come about as a result of sin. Redeemed mathematicians must refrain from participating in such activities and should actively combat them. Christian mathematicians should also not yield to the temptation of self-aggrandizement through mathematical activity. Desire for fame or fortune should not motivate Christian mathematicians;

they should do their work to glorify God, working honestly and to the best of their ability, but above all, in humility.

Christian beliefs and principles come to the fore in the teaching of mathematics, too, because a personal relationship between teacher and students is involved. By their example and by the way they treat students, teachers can put into practice their Christian beliefs.

In all these cases, Christianity clearly impinges upon mathematical practice. Christian principles are thought to have little or nothing to say, however, regarding the content or theory of mathematics. Here this view approaches the first one - the field of mathematics proper is religiously neutral. That mathematics has anything to do with the Christian religion at all is due to the fact that mathematicians are human beings and so ought to adhere to Christian norms for conduct.

#### THE "TWO REALMS WITH INDIRECT INTERCONNECTIONS" APPROACH

A third type of approach which is often adopted by Christians as a supplement to the approach just described is one in which a kind of indirect cross relationship is asserted. I will distinguish two main types of approach within this class, depending on which realm is seen as influencing the other. Some Christians seem to adopt one of these to the exclusion of the other, while others embrace them both.

There are first of all those who wish to stress the relevance of mathematics for the Christian religion. Here we encounter mainly the phenomenon of Christian apologetics. While such people may also hold to the viewpoint I will describe next and so will gladly give a testimony regarding how mathematics fits into God's overall plan for the world, they generally think they have more important things to use mathematics for; namely, to witness to their unbelieving brother and sister mathematician. This can be done best, they feel, by entering the unbeliever's own territory and arguing on his or her grounds. In this way any points they score will have to be conceded by their opponent.

The role of mathematics in such an enterprise is usually that of demonstrating the respectability or the reasonableness of the gospel. Mathematics is usually taken as a model for Christian faith and doctrine. Analogies are drawn between the method of mathematics and the nature of Christian belief. Christians believe doctrines purely on faith, but this is no different from what mathematicians do when they accept their results on the basis of axioms. What's more, according to Gödel's incompleteness theorem, mathematics can't even prove all of its true results on the basis of any manageable system of axioms. Christianity is therefore no more irrational or subjective or belief-ridden than mathematics is. In this way Christian apologetics attempts to lead the unbeliever to the

point where he or she is willing to consider Christianity on its own merits, the stigma which attaches to Christianity having been cut loose by the mathematically based argumentation.

In addition to the methodology of mathematics, mathematical results may also serve as ammunition for apologetics. Mathematicians accept certain results and ideas, say, in transfinite set theory, even though they cannot fully understand them. Christians are thus no odder than mathematicians when they accept dogmas they cannot completely comprehend. The idea of the Trinity or of God's incarnation in the man Jesus Christ may be accepted as logically consistent by analogy with some particular mathematical result.

The intent here, as before, is not to use mathematics as a model for developing systematic theology, but to wield it as a tool for Christian apologetics with respect to certain doctrines. Apologetics is seen to be a propaedeutic that makes use of religiously neutral ideas in order to lead men to be open to the supernatural truths of the Scripture.

A second category within this general approach looks at how the Christian faith may affect mathematics, broadly conceived. Many Christian mathematicians feel that their Christianity must be relevant in some way or other to their work in mathematics. They are not content merely to apply Christianity to the arena of personal interaction between mathematicians and others; yet they cannot conceive of it as transforming the methods or results of mathematics. According to this approach, mathematics is usually seen as a realm under God's common or universal grace. All men, regardless of religious persuasion, are permitted by God to discover mathematical truths, which are exactly the same for everyone. Religious differences enter into the picture only when the origin and ultimate purpose of mathematics is considered. Christians recognize the final source of mathematical truth, as of all truth, to be God, while unbelievers blindly close their eyes to this fact, perhaps even deluding themselves by imagining that they invent mathematical ideas and results by the power of their own minds. Christians see God's handiwork everywhere in the world of mathematics, in its perfect logical consistency, in its aesthetically pleasing structure, and in its uncanny applicability to the physical world. Such design points directly to a divine author and should inspire devotion to God. God is the Supreme Mathematician, the one who knows all mathematical results perfectly. Human mathematicians, being created in the image of God, are able to think God's (mathematical) thoughts after him, albeit imperfectly, due to the finitude of the human mind. Depending on how such Christians view the nature of mathematics, some may go further and view God as the Supreme Mathematical Architect, making use of his full knowledge of mathematics to construct the universe, for as they see it, the laws by which the physical world is governed are essentially mathematical laws.

## INTERLUDE

Before proceeding to discuss the last view, I'd like to pause to make a few qualifying comments about the above analysis to avoid being misunderstood. In the first place, each position I've described is somewhat amorphous. My classification categorizes major emphases roughly according to the type of relationship involved. I think this tack has some merit as an initial analysis of the various positions, but I admit that it still glosses over some significant differences in outlook. A particular feature of one position may be fervently espoused by some adherents and yet be dismissed by others. Someone may tend toward one of the above positions without holding to everything I mentioned under it.

In the second place, it should be clear from my exposition of the various positions that they are not necessarily in opposition with one another. I've attempted to identify main tendencies, not partition the set of Christian mathematicians into so many disjoint classes. There may very well be Christians who feel that an external relationship is all that there is, but others may wish to be Christian mathematical apologists as well. The type of approach in each option is different, but to some extent it is possible to take them as complementary, so long as none of them are taken to excess.

A consistent approach might be developed, I suppose, which blends together elements from more than one approach, care being taken to avoid or modify any extreme tenets which could not be harmonized with others. In order to attain a unified approach in relating Christianity and mathematics, then, we might attempt to find the greatest common denominator of all the different approaches (beyond the first one, of course, for it definitely contradicts the following ones) and form them into a grand synthesis that incorporates key features of each position taken in moderation. Combining these approaches in this way might be taken as analogous to joining together the various descriptions given of an elephant by a group of near-sighted people, each of whom can see only a limited part of the animal. The different approaches could thus be seen as specialized perspectives, each one zeroing in on the same issue from a different angle. On the surface, this strategy would seem to be a wise one for Christian mathematicians to follow, for it doesn't make much sense to hold to a position that ignores connections that really are present while exaggerating other ones all out of proportion.

Rational as it might appear, I do not think that it is a viable solution. For one thing, each approach would need to be modified and certain elements chosen from it to combine with elements from the other approaches. What criteria should be used to choose these elements? How should the various approaches be softened? Different people favor different aspects of these approaches and would undoubtedly campaign for their retention, perhaps even emphasis, in the resulting synthesis.

In addition to these complications, there is another drawback. For one thing, we are still leaving out the possibility that a more direct relationship between mathematics and Christianity might exist. Presumably we could continue by adjoining this approach and any other one as well, and so obtain a grand synthesis of all of them except the very first one. But on closer inspection, I think this composite alternative will be seen to be impossible. This final approach will seem to many to be an extreme position which cannot be moderated and so cannot be genuinely combined with the above approaches. Moreover, were such a synthesis attempted, I think it would get us no closer to a common understanding of the issue, but would only continue to cover up differences in outlook that are real and important. For example, the more direct approach which I will be discussing next and to which I hold begins by explicitly rejecting an assumption that all the other positions take for granted. As I see it, each of the approaches mentioned up until now share a common starting point insofar as they accept some sort of two-realm formulation of the problem. In my opinion this is a defect which in the long run will prove fatal to the development of a distinctively Christian philosophy of mathematics, so I cannot align myself with any combination of these approaches. There will naturally be similarities between my view and the ones I've already described, but I believe it will become clear as I explicate the following position that there is a major difference in outlook between it and the other positions.

#### THE "ONE REALM WITH CENTER AND SECTOR" APPROACH

My own approach begins, as I have just mentioned, by calling into question the way in which the problem is posed. There are not two realms, mathematics and the Christian religion, which must somehow be related or integrated. To be a Christian is quite a different matter from being a mathematician. These two callings or vocations are on different planes, as it were. I am not a mathematician, for instance, when I am baking bread or when I am playing volleyball or when I am singing a bed-time song to my children. I do not deny that I continue to have certain mathematical talents when I am doing these other things, nor that they may influence the way in which I engage in them - one part of my life affects the rest - but I am not functioning as a mathematician when I am doing them. Mathematical activity is only one sector or dimension of my life along with many others. This is not the case, however, with my being a Christian. As a Christian I am called to be a follower of Christ in whatever I do, so that all aspects of my life may glorify God.

Christianity is thus not a dimension of life or a specialized calling or a separate realm at all; our Christian faith must permeate everything we do. To use a metaphor, our heart commitment to God lies at the center or hub of our being, giving religious direction and motivation to the whole wheel of our life's activities, one sector of which, for those of us who



are mathematicians, is doing mathematics.

Viewing things in this way, I cannot escape coming to the conclusion that our mathematical activity, too, is called upon to be transformed by the renewing of our mind; that it, too, must be taken captive and made obedient to Christ. This speaks to me of a more direct relationship holding between mathematics and religious belief than is envisioned by the positions I've discussed so far. God's Word definitely gives norms regarding interpersonal relationships and tells us how we are to treat our neighbor, but to stop there is to lop off an area of life and call it religiously neutral, untouched and untouchable by either sin or saving grace. I cannot reconcile such an approach with my understanding of Scripture. All things are under the Lordship of Jesus Christ. On the broad issue of Christ and culture, I stand in the line of transformational Christianity which extends from St. Augustine and John Calvin through such Dutch Calvinistic thinkers as Abraham Kuyper, Herman Dooyeweerd, and Dirk Vollenhoven of a generation or two ago, and which is found today in several North American educational institutions, the foremost of which is the Institute for Christian Studies in Toronto. Because of my stand on this more general issue, I am committed to trying to work out this type of approach also with regard to the relation of Christianity and mathematics.

The position which I would like to see developed, then, assumes that a more direct relationship between Christianity and mathematics is called for. Very briefly, the relationship may be described as follows. The Bible gives us our basic orientation toward life, telling us who we are, how we fit into the cosmos, and how we are to treat our fellow human beings; who God is, and how he deals with his creatures; who Christ is, and how we may be saved from our sin; and so on. On the basis of what God reveals of himself and his creation, we can begin to form a more or less coherent picture of the world. This is still subject to distortion because of sin, but in principle God's revelation makes possible a Christian world-and-life view. Through systematic communal reflection upon and concrete analysis of created reality in the light of God's Word, our basic outlook on the world can be refined or developed into a Christian philosophy. Such a philosophy will continually interact with each of the special sciences, including mathematics, to discover what the proper domain of study is for that science, what methods are particularly suited for its task, and how these are related to what the other sciences investigate and how they proceed.

In this semi-direct way, Christianity comes to bear upon mathematical theory and practice. Christian mathematics, according to this view, is not a matter of adding souls or subtracting sins, nor is it a matter of attaching spiritual significance to certain numbers and shapes. Christian mathematics is not so much a matter of special subject matter and methodology, of peculiar facts and techniques known only to born-again believers, as it is a matter of religious color and

total perspective. Life cannot be atomized into watertight compartments, mathematics over here and Christianity over there; it is an integral whole. Only by adopting a holistic viewpoint will we be able to see how Christianity impinges upon mathematics in the way I am suggesting. We are not called to integrate our faith with our mathematical work, but to work out our faith in our mathematics.

If our view of mathematics is broadened, perhaps we can also start to see in what way even the results and methods might differ for Christians and non-Christians. Isolated facts and individual techniques of mathematics do not exist apart from a broader context giving them their meaning and value. Mathematics is more than a body of facts and methods; it is an activity done by human beings in certain historical settings, and this influences the sorts of theories and techniques which are developed.

To illustrate this, let's take the example which is invariably brought up by those who cannot fathom the possibility of the sort of direct relationship between Christianity and mathematics that I am advocating. "Within the system of natural numbers," they counter, "doesn't  $2 + 2 = 4$ , regardless of one's religious or philosophical outlook? Isn't this elementary proposition a religiously neutral fact that all people everywhere can accept?" The answer seems to me to be: "Yes" and "No". Yes, the same thing happens whenever anyone, regardless of his or her religious or philosophical outlook, takes a collection of 2 apples and adds 2 more to it; there will be 4 apples in all. Everyone can depend upon this to be so because God continues to maintain the creation in its numerical functioning according to his laws for arithmetic. This is what I understand common grace to be: God remains faithful in upholding his creation. But this is not yet mathematics. Mathematics begins when people respond analytically to such occurrences, when the regularity underlying events such as these are recognized and theorized about. Mathematics goes beyond computation, though computation - and even naming or symbolizing numbers in a systematic way - presupposes some implicit understanding of mathematical ideas and laws, if not a worked out theory of arithmetic. When the "fact" that  $2 + 2 = 4$  and other "facts" of arithmetic are put into a theoretical system, then one is doing mathematics. It is difficult to circumscribe just what mathematics (arithmetic, in this case) must consist of, since there are such different levels of comprehension and formalism present in the arithmetics of, say, Fibonacci, Peano, and Russell, but all arithmetic attempts to give a systematic account of numerical behavior. It is thus embedded in a meaning-framework which includes, but is not confined to, the system of mathematics developed around the known facts. On this level, it is rather easy to show that answer to the original question is "No":  $2 + 2 = 4$  is not the same for everyone. No normal adult will deny that  $2 + 2 = 4$ , but when mathematicians and philosophers of mathematics begin to spell out what they mean by this statement, it soon becomes apparent that

they mean widely different things by it. (Incidentally, one of the things they do not mean by it is that 2 apples and 2 apples are 4 apples.) A logicist such as Frege or Russell would see in this sentence a proposition of logic which, when unraveled, involves sets or classes, one-to-one correspondence, set union, and logical identity. Strict formalists would see no meaning in this sentence at all. For them there is only a well-formed string of symbols that can be rigorously derived by means of accepted rules of logical inference from other strings of symbols that are themselves meaningless but which have been designated the axioms and definitions of the system of arithmetic. Intuitionistic mathematicians would see a third meaning in this statement, a Platonist perhaps a fourth thing, and so on.

By now some will be protesting, "That's philosophy of mathematics, not mathematics pure and proper!" I don't disagree that it is philosophy, but I don't think mathematics can be totally separated from its philosophy. Different schools of mathematics are a historical fact; one does not avoid membership in them merely by ignorance. Mathematicians cannot completely suspend judgment on what they are doing in their work, how it fits into the encyclopedia of knowledge, and what its relation to reality is. Mathematicians need not possess a worked out philosophy of mathematics in order to reflect a dominant viewpoint. Without being fully aware of it, they may exhibit certain philosophical tendencies because of others who have had an influence on them or even because of the historical way in which the branch of mathematics in which they work has unfolded.

Moreover, though my comments regarding the meaning of the statement " $2 + 2 = 4$ " touch only upon philosophy of mathematics, showing that there is difference of opinion on that level, this does not mean that on a technical mathematical level there are no differences between the various schools of mathematics. Different philosophical positions do lead to different developments of mathematics; priorities and at times even technical results differ. The logicist program of mathematics may lead to a different development of a mathematical topic than that preferred by the formalist or the intuitionist. To take an example, the grandiose "genetic" approach to developing the real number system ultimately from set theory fits in rather well with the logicist approach to mathematics, but a formalist might prefer a less ambitious, axiomatic approach, "defining" the real numbers by means of a number of postulates which describe the real number system as a complete, ordered field. Constructivist or intuitionist mathematicians, on the other hand, would not be happy with either approach to the real number system; for them the classical concept of a real number itself is problematic and needs to be completely rethought.

Now, I have no alternative development of mathematics or even of real analysis to propose in opposition to those of the logicist, formalist, and constructivist, nor do I think a Christian should reject the various approaches out of hand. All

I want to argue for here is that philosophy of mathematics does have an effect upon the technical development of mathematics, both in its organizational set-up and in its formal results. I therefore believe that if a fully developed Christian philosophy of mathematics gets articulated, it too might affect the priorities and the formal results of mathematics, both positively and negatively. At least I see no reason why such a possibility should be ruled out at the start as absurd. Since a Christian's belief guides his or her philosophy of mathematics, it seems to me that one must be open to a more direct relationship existing between Christianity and mathematics than that envisioned by the earlier approaches we described.

Given the above conclusion, that one's religious and philosophical presuppositions influence one's view of mathematics, we are confronted with a different set of problems than that faced by the proponents of the other approaches. A major issue which needs to be addressed is this: if there is diversity within mathematics, why is there basically only one mathematics taught in schools and universities and published in mathematical journals? Also, what attitude should we Christians have toward it? Are we called upon to reject it and start all over with our own system of mathematics?

In answer to the first question, we should note that there are alternative approaches within mathematics, even though they are not usually billed as such. Comparison of the way in which certain topics are developed in different mathematical textbooks shows different philosophies of mathematics at work. That this variety is not emphasized and is not very well known to students or even some teachers of mathematics only attests to the unspoken practice of absorbing different mainline approaches to mathematics as if they were complementary to one another, useful for different purposes or subject matter. Those that don't quite fit, such as the constructivist approach, are ignored or banished. In any case, however, the overwhelming predominance of one particular philosophical viewpoint in developing a given topic is certainly no argument in favor of there not being any at all.

In response to the second question, I believe our attitude toward existent mathematics, just as toward any other area of scholarship, should be one of critical evaluation and reformation. Since the mathematics we know has developed in Western civilization, where Christianity has been an active force, even if not the only one, it may be that mathematics still shows characteristics which derive from a Christian source. But even where this is found not to be the case, or where the approach has been secularized or is judged to be anti-Christian, we are still not licensed to jettison the mathematics. Unbelievers may distort what they see because of a warped religious and philosophical outlook, but it is still something in God's world which they are accounting for. Our task, then, is the more difficult one of taking their insights and remolding or

reforming them. We must ask ourselves, "What in the world is it that is being looked at here? How do we see these matters from our perspective?" In this way we will be led to reinterpret their work, to put it into our own Christian framework.

In talking about the relationship between Christianity and mathematics, I have remained somewhat abstract, arguing merely for consideration of the possibility that one's Christian outlook on the world might affect his or her work in mathematics proper. I will now proceed to exhibit the main lines of that relationship as I see it. The relationship which I believe to be correct is a direct one: the Christian religion influences mathematical work through the mediation of a Christian world view or philosophy of mathematics. A Christian mathematical system or philosophy of mathematics is not deduced from the Bible or from some set of theological propositions. Rather, a Christian approach to mathematics results by analyzing the meaning of created reality and mathematical practice in the light of Scriptural themes and directives. Through the working of the Holy Spirit we can come to understand the implications of God's Word for the contours and boundaries of our approach. Let me now be a bit more specific and briefly sketch what some of these seem to me to be.

#### BIBLICAL FOUNDATION OF A REFORMATIONAL APPROACH TO MATHEMATICS

The central thread running through the Scriptures is that of creation, fall, and redemption. The Bible is often looked upon as the story of salvation, which it is, but salvation only comes about in the context of a fall into sin. Sin and salvation by themselves are still not sufficient for describing the whole message of the Bible, because neither one can be understood apart from what God tells us about creation. For sin affects more than just one or two dimensions of our life; it has its effect on the world as a whole. And salvation is more than saving souls and putting people back in touch with God; it is in principle a restoration or renewal of all creation. So then to summarize the good news of God's Word, we must begin at creation, as it also does.

From Genesis and elsewhere in the Bible, we learn many things about the creation of the world. We learn that it was Yahweh who created all things by his Word, and that they were made very good. By his powerful Word, all creation was made and continues to be upheld. We also learn that God made different kinds of things, having different functions within the world. A rose is a rose, not a stone or a snail. The particular tasks given to men and women were to be fruitful and multiply and to exercise dominion over the creation, to take good care of it and develop or realize its potential. Human beings were not God, but they were creatures made in his likeness.

In their apostate rebellion against God, though, Adam and Eve wished to elevate themselves to divine preeminence. They

were no longer content to be creatures, but wished to be like God. As a result of the fall into sin, their relationship with God was strained to the point of breaking. Sin also caused hate to mar the relationships between people. Even the ground was cursed on account of sin. Creation was no longer the harmonious whole that God had made. Nevertheless, God would not let his creation go. He promised to redeem it, to set it straight again through the seed of the woman, his only son, Jesus Christ.

In the interim, God himself took drastic means to maintain his creation. At one time he drowned the world with a flood, saving Noah, his family, and a floating zoo of all kinds of animals so that a new beginning of sorts could be made. Later he chose a man, Abraham, and his descendents, Isaac, Jacob, and their offspring, through which to work out his promises. Though his chosen people Israel rebelled against him time after time, he continued to rescue and preserve at least a remnant of them, always pointing them back to the norms for the good life. Earlier, at Sinai, he had spelled out in great detail exactly how life ought to be lived. Loving God and neighbor meant sacrifices for sin, obeying your parents, being faithful in marriage, not slandering your neighbor, providing for the poor and helpless, and so on, but it also meant caring for the animals and giving the land its proper rest. God's shalom meant peace between God and his people and between neighbor and neighbor, but it also meant that one day the lion and the lamb could lie down peaceably together.

Salvation came for the creation through Jesus Christ, the Word of God made flesh. He fulfilled God's law perfectly, offering himself as the Passover Lamb in payment for sin. Throughout his ministry on earth he preached and demonstrated to the crowds what the coming kingdom of God meant: the sick and handicapped were made whole, demons were driven out, food was provided for the hungry, the dead were raised, and so on. All that was required of men and women was true repentance, to love God above all and their neighbors as themselves. After Christ's death, resurrection, and ascension, his Spirit-filled followers brought the message of the kingdom to those around them, first to the Jews, but also to the Gentiles. Those who heard the Word and believed in their hearts that Jesus was Lord were added to the new peoplehood, Christ's body. To that church of God we also belong today if we confess his name.

I have already spelled out above some of the implications of this Biblical theme for mathematical and philosophical activity. My approach to philosophy of mathematics and to the topic of the relation of Christianity and mathematics is consonant with what I believe the Bible says about fall and redemption. Both sin and salvation are radical, both affect life in its entirety, including academic and scientific work. Hence my perspective regarding the necessity of a Christian approach to the content of mathematics, of an inner reformation of its philosophical foundations. What I would like to focus on here, then, is

primarily the first part of this theme: creation. What does the Biblical teaching about creation have to say about our view of mathematics?

All things have been created and are held together by God, the Sovereign Ruler of the world. Positively, this credo means that God structures the cosmos we experience, making things what they are. Human beings are not the law-givers for reality, whether directly or by means of models; they merely experience and search out lawful regularity in the ways things function all around them.

Negatively, God's sovereignty and the Creator / creation distinction means that nothing creaturely is ultimate or substantive or absolute. All things are alike subservient to the Will of God. It is therefore religiously wrong to elevate something in the creation to a position of all-importance, whether an object or a relationship or an aspect of life. One part of the creation cannot give ultimate meaning to another part; they are all interrelated, relative parts of a whole which exists ultimately for the glory of God and which finds its full meaning in Christ and his Kingdom.

The implications of these ideas for a Christian view of mathematics are the following. Mathematics deals with created reality, with aspects of a world structured by God's Word. The exact scope of mathematics' subject matter is something that must be determined a posteriori by analyzing reality and mathematical activity, but that people do not invent mathematics as a free mental creation or merely play games of symbol manipulation, I take to be a basic Biblical presupposition of any Christian philosophy of mathematics. This does not mean that men and women are passive recipients in doing mathematics; quite the contrary. They are active in many different ways as they develop mathematical theories. Mathematical ideas are not read directly off an external world, each concept having a sensory counterpart, but arise through genuine human participation. It also does not mean that ideas may not be played with to see what their consequences might be, nor that opposing axiomatic systems may not be entertained or adopted. But this is never purely an arbitrary game. A mathematical system or idea can always be traced back in some way or other to a real-life source, and it must eventually find its reason for existence in helping people better understand and care for the world around them.

A second implication which I see is that mathematics is of limited scope and significance; it is not the queen of the sciences. Since it only concentrates on certain aspects of real situations, its practitioners should not absolutize it as the source of certainty and truth. Our western world has had a long and deleterious tradition of deifying all that is considered strictly scientific, where science is patterned after mathematical physics or is at least cast into a mathematical mold, statistical or otherwise. If something can't be quantified

or measured, there is a strong tendency among the intellectual leaders of western culture to ignore it or to dismiss it as worthless. Prediction and control are everything in determining policies, and they are fueled by numbers and formulae. Christian mathematicians should speak out against this trend, for meaning and certainty do not derive from and cannot be guaranteed by mathematics.

The specialized character of mathematics has implications for applying mathematics; it also relates to defining the scope of mathematics. The exact scope of mathematics and its relationships to other areas cannot be stipulated in an a priori manner, but one must be wary of reductionism. This applies to attempts to push out the boundaries of mathematics to include other fields, but also to attempts to reduce mathematics to some other field of thought, such as logic, and even to attempts to treat mathematics internally in a monolithic fashion, such as was tried in the trend toward arithmetization in the 19th century or in the trend to reduce everything to set theory in the 20th. The watchword here might well be taken to be "complementarity", as Willem Kuyk has done. There is a richness to created reality which should not be denied theoretically by forcing fields into a hierarchy, subsuming some fields under others which in turn have been elevated to heights of all-importance.

To summarize my position, then, I feel that a Christian outlook on mathematics must acknowledge it to be a special science dealing with certain aspects of created reality. It is this perspective which I think should inform our theory and practice of mathematics. To take just two particular examples, I think it should underlie our philosophy of mathematics and our approach to mathematics education. I do not have the time or the ability to develop either of these topics in elaborate detail, but I will make a few closing remarks about each of them.

#### A REFORMATIONAL APPROACH TO PHILOSOPHY OF MATHEMATICS

"Mathematics" is a collective term which denotes a group of related fields of thought. The various branches of mathematics all find their roots in human knowledge of the numerical and spatial dimensions of the world, of discrete and continuous phenomena. While these aspects do not exhaust the ways created reality functions, they can be found everywhere. All things have numerical and spatial components, and can therefore be approached mathematically. Thus, in a certain sense, mathematics' range of applicability is as broad as the world, though it still studies things from a limited perspective. By focusing on the numerical and spatial properties of things, people can form certain concepts, demonstrate their logical interrelationships, draw out their consequences, put them into an axiomatic system, and treat a family of similar ideas in a very general setting. Yet the bedrock upon which all this activity takes place is the numerical and spatial functioning of real



things.

Mathematics is at the same time a fully human product. Abstraction, idealization, induction, analogy, intuition, generalization, deduction - all these and more are at work in the formation of mathematical ideas and theories. Mathematicians do not invent ideas in an arbitrary way, but they also do not simply discover them or copy reality. I do not believe that mathematical ideas exist, for instance, in some Platonic mental realm, awaiting discovery. I also think it is highly speculative to say that God does mathematics or that he made use of mathematical notions in creating the world or that his providential maintenance is dependent upon results from some branch of mathematics. The only mathematics we know is that constructed by human beings, having a history to it which reveals a variety of religious and philosophical outlooks as well as a more formal, technical development and varied application of its results. Which mathematics might God's be: classical Greek mathematics? Renaissance mathematics? 19th century mathematics? Bertrand Russell's? Hilbert's? Bourbaki's? Bishop's? We are on much safer ground if we refuse to be seduced into such conjecture, even though it promises to tie Christianity and philosophy of mathematics quite closely together. Our philosophy of mathematics should be earth-bound, though consonant with our belief in God and his Revelation.

If we were to test the dominant philosophies of mathematics by means of these germinal ideas to see whether they harmonize with our general outlook, I believe we would soon find that we are in basic agreement with none of them. What is needed, therefore, is the development of a distinctively Christian philosophy of mathematics. Some work has been done along these lines by people who share my basic approach and who might be classified (as was done in our BIBLIOGRAPHY) as Calvinistic or as belonging to the Amsterdam school of reformational, Christian philosophy, but much more is still waiting to be done, especially in a way that will communicate clearly with ordinary Christian mathematicians, not to mention secular mathematicians.

#### A REFORMATIONAL APPROACH TO MATHEMATICS EDUCATION

With respect to mathematics education a Christian approach is also possible, one which extends beyond the realm of interpersonal ethics. A Christian philosophy of mathematics quite obviously affects how one approaches the subject matter to teach it. The key idea here can perhaps be summed up best by saying that the study of mathematics should be reality-oriented. Children should be continually reminded by the structure of their studies that they are dealing with God's world, that mathematics originates in human experience of numerical and spatial states of affairs. This is especially important for elementary school mathematics, where children are apt to get lost if they are taught abstract concepts or mechanical techniques. I'm convinced

it applies to mathematics education on all levels, but because I have had first hand experience at the Curriculum Development Centre in Toronto in constructing a mathematics curriculum for the primary grades, I would like to focus my remarks on the elementary level. Parallels to other levels of education definitely exist and can be made where appropriate.

In saying that mathematics should be reality-oriented, I'm not thinking first of all of applications. Applications are often used merely as illustration or practice for some given problem-solving technique or abstract concept. Children aren't fooled by this tactic for very long. They soon learn as they jump from one problem to the next one totally unrelated to it that only the mathematical idea or method is important; the real-life context counts for almost nothing. They thus look to examples as mere prototypes for working their problems rather than learn to think mathematically about real-life situations. Memorization and thoughtless, automatic responses are thus de facto what is being taught rather than understanding of concepts and mathematical analysis of situations. To encourage the latter, children should begin with situations and problems that originate from their own concrete daily experiences. This will provide a rich environment in which to learn mathematics, for mathematics is all around us, if only we have eyes to see it. Time should be taken to investigate a topic in a more extended and holistic way so that the children can learn the mathematical ideas and procedures connected with it as best they can. In order to guide the learning process, teachers may have to learn how to deal with mathematics in a more integral way than they usually do, for we are all too used to having life in neat, little compartments. As children study a given theme or topic from a variety of different angles, numerical and spatial properties and problems will arise and call for recognition and exploration. Teachers trained to think about mathematics in a reality-oriented manner will be able to help the children focus on certain aspects of a topic. Children will thus learn from experience that mathematical ideas develop out of real situations and that they contribute their important but limited share to the aggregate understanding of the topic.

Introducing mathematical ideas in this broad, thematic way will allow children to gain some elementary intuition about them, to play with the concepts in a less formal and formidable setting, and to forge some of their own techniques. I think this is quite necessary and important, but it is clearly not sufficient. Children must also be given exposure to mathematics in a more systematic way. Mathematics has a theoretical structure of its own, certain complex ideas being built upon other simpler ones. Mathematical knowledge can be conceptualized most efficiently when one pays attention to this order of complexity. There is admittedly some degree of latitude possible here - certainly more than is usually acknowledged by any given textbook - and a strictly logical approach to a topic need not be pedagogically wise. Nevertheless, it makes sound educational

sense to build up a program by utilizing at each stage what has been learned earlier. Doing all this, however, one must still maintain a reality-oriented approach, even here. Real life problems must form the motivation for a topic and should be an important part of the children's practice. Besides using "applications" before and after the ideas are introduced, concrete materials such as counters or grouped sticks or shape-blocks should be employed in the very process of fixing the concepts and techniques. These should no longer be embedded in a broader thematic topic and so will be somewhat "abstract" or context-free, but their tangible concreteness will help children to grasp the ideas involved and gradually move toward a more conceptual understanding of mathematics.

## CONCLUSION

With these brief comments on philosophy of mathematics and mathematics education I will conclude. I have only begun to scratch the surface of what this last approach could mean. More could still be said, and beyond that there is much that requires further research and communal discussion. My intent here, though, is not to present a finished product which can be evaluated and either accepted or rejected, but to encourage ongoing reflection on the topic of Christianity and mathematics, and to present an alternative to the approaches I analyzed earlier, one that I believe is more in line with the central message of Scripture. If the approach I have outlined fails to gain wide acceptance by other Christian mathematicians, I hope that my analysis and arguments will at least promote a deeper discussion of the possible causes behind the diverse approaches to the topic of Christianity and mathematics. If, on the other hand, this outlook is accepted as basically sound, I would hope that a circle of Christian mathematicians would develop this general perspective into a full-fledged philosophy of mathematics and draw out its implications for mathematics education and mathematics itself. The detailed reformation of the philosophy of mathematics has yet to take place; there is plenty of work to be done before it will become a viable option in the market place of mathematics. I pray that in the coming years God will give us the insight and ability we need to work out such a vision in a program that will affect all facets of mathematics.