# INFINITY & REALITY

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Anyone who has ever tried to discuss topics such as 'Infinity' or 'Reality' knows something of the difficulty of the task undertaken. To try to cover both of these topics in thirty minutes is well nigh impossible. But because of the relevance of these topics to the theme of this conference, I would like to try to at least suggest a possible relationship and see where it might lead us in the subsequent discussion. Furthermore, Robert Browning has said "...a man's reach should exceed his grasp, else what's a heaven for?" So here goes.

In April of 1976 an article appeared in The American Mathematical Monthly by Felix E. Browder entitled "The Relevance of Mathematics." He states that several years earlier he had been asked to give a lecture to an audience of undergraduates who were somewhat limited in their knowledge of mathematics. He chose to ask the question "Is mathematics relevant, and if so, to what?" He notes that both the terms 'mathematics' and 'relevant' have a vagueness associated with their use that demands clarification. Concerning 'relevant' he notes that "...the modifying clause, 'and if so, to what?' points to the basic vagueness of the customary usage of 'relevant' by asking implicitly: What body of values or purposes? We know that different bodies of values or purposes have been emphasized or pursued within different social or historical contexts, on the basis of different perspectives of the important aspects of the human condition."

He questions whether we have reached any consensus as to what is 'Good' and whether we can tie together all the diverse views into any coherent view of the human condition. He then notes that his lecture is in a way a response to the public demand for relevance as it relates to mathematics. He begins with Plato and quickly concludes that "Plato's solution of the problem of value by identifying the Good with mathematics is one that very few of us nowadays would be willing to defend, especially in public." However, he notes that "Mathematics itself has been consumed over the past hundred years as well as pricked, delighted and tormented by Cantor's great Platonic vision of the theory of the accomplished infinite, the precise reasoned theory of infinite magnitudes." He then proceeds to define Mathematics I, II, III, and IV which very briefly are as follows:

Mathematics I: Computational techniques, counting, measuring and calculating.

Mathematics II: Concepts and techniques to formulate and solve problems in other intellectual disciplines.

Mathematics III: Research--"a creative art which works on the objective material of given problems and concepts by means of inventive jumps and intuitions."

Mathematics IV: "...the vision of mathematics as the ultimate and transparent form of all human knowledge and practice."

"...a vision of mathematics as the total science of intellectual order, as the science of pattern and structure."

He cites Alfred North Whitehead as saying:

> "The notion of the importance of pattern is as old as civilization. Every art is founded on the study of patterns. The cohesion of social systems depends on the maintenance of patterns of behavior, and advances in civilization depend upon the fortunate modification of such behavior patterns. Thus the infusion of patterns into natural occurences and the stability of such patterns is the necessary condition for the realization of the Good. Mathematics is the most powerful technique for the understanding of patterns. Here we reach the fundamental justification for the topic of Plato's lecture (Lecture on Good). Having regard to the immensity of its subject matter, mathematics, even modern mathematics is a science in its babyhood. If civilization continues to advance, in the next two thousand years, the overwhelming novelty in human thought will be the dominance of mathematical understanding."

In this paper we would like to try to utilize this pattern approach to some ideas which we often avoid because they are so difficult to discuss in terms of everyday experiences.

David Hilbert has been quoted as saying "From time immemorial, the infinite has stirred men's <u>emotions</u> more than any other question. Hardly any other <u>idea</u> has stimulated the mind so fruitfully. Yet, no other <u>concept</u> needs <u>clarification</u> more than it does." (2) Hermann Weyl has said that the phrase that might well characterize the life center of mathematics is "Mathematics is the science of the infinite."

In this paper we wish to get first some historical perspective on how men have viewed the infinite. We want to see how this concept has changed and matured to the point that it is a useful and necessary idea on which much (if not all) of our present scientific knowledge is based. We cannot go into the history of infinity in any great detail, but a brief look at how men have thought about it will put the rest of what we want to say in some perspective. Then we also want to take a brief look at 'reality'. What do we mean when we use this term? What relationship, if any, does it have with infinity? We want to see if perhaps we might use infinity to help explain or understand reality. We would also like to try to put these two concepts into a religious perspective. Someone has said that a man's religion is that on which he bets his life. Men are incurably religious and there is no reason why we should not try to see if there is some relationship between the kind of thinking we do in mathematics and the religious commitments we make.

No apology is needed to discuss mathematics and religion. There is historical precedence for relating the two. Dantzig says,

> "Kepler reluctantly engaged in astronomy after his hopes of becoming an ecclesiastic were frustrated; Pascal gave up mathematics to become a religious recluse; Descartes' sympathy for Galileo was tempered by his faith in the authority of the church; Newton in the intervals between his masterpieces wrote tracts on theology; Leibniz was dreaming of number schemes which would make the world safe for Christianity." (3)

With our increased knowledge today about how the human mind operates and the vast scientific productivity that has blossomed from reasoned thought what we need very much today is more interplay between the disciplines and a reasonable effort at some kind of coherence concerning the human condition.

I am aware that mathematical arguments have been used before to try to prove theological beliefs. I believe it was Augustus Demorgan who has been quoted as saying, "When a very young man, I was frequently exhorted to one or another view of religion by pastors and others who thought that a mathematical argument would be irresistible. And I have heard the following more than once..."

> Since eternal happiness belonged to the particular views in question, a benefit infinitely great, then, even if the probability of their

> arguments were small, or even infinitely small, yet the product of the chance and the benefit according to the usual rule, might give a result which no one in prudence ought to pass over."
>
> (Here we are referring to the law of mathematical expectation.)

Now I do not intend to prove anything in this paper. A quotation attributed to John Dewey and which sounds very much like him but which I have been unable to document is as follows:

> "Mathematics is said to have disciplinary value in habituating the pupil to accuracy of statement and closeness of reasoning; it has utilitarian value in giving command of the arts of calculation involved in trade and the arts; cultural value in its enlargement of the imagination in dealing with the most general relations of things; even religious value in its concept of the infinite and allied ideas. But clearly mathematics does not accomplish such results, because it is endowed with miraculous potencies called values; it has these values if and when it accomplishes these results and not otherwise."

Note in passing the phrase "religious value in its concept of the infinite and allied ideas" which we want to enlarge upon later. Rather than prove anything we want to utilize what we have learned and how we have learned it in mathematics to help us cope with two difficult ideas--infinity and reality.

Cassius J. Keyser in a lecture entitled "The Role of Infinity in the Cosmology of Epicurus" says

> "In any adequate historico-critical survey of the role which the notion of infinity has played in our human thinking, the thought of many thinkers, widely distributed in time and space, would have to be passed in review --analyzed, understood, and appraised. Among the questions which the critic would have to ask and try to answer respecting each thinker are such as these: What did he mean by infinity? Did he employ the term to denote a definite concept or at best a vague and emotional intuition? Were his thought and use of it mystical, or logical and analytical, or both? Did he regard his infinite as a fact

> or as an hypothesis, and why? Was it time? An extension of time? Space? An extension of space? Was it matter or mind or both? Was it physical or spiritual? Concrete or abstract? Did he define it and, if so, did he do it consciously? Did he think of it as magnitude or as multitude, or both? Had he but one infinite or many of them? If many, were they coordinate or hierarchial? If the latter, was the hierarchy crowned or summitless? Was his infinite subordinate in his thought or central and dominate? Did he employ it consistently or confusedly? Was its function poetic or scientific or both? What was its relation to the modern concept of mathematical infinity?" (4)

Obviously such an 'adequate historico-critical survey' is impossible here. Jose A. Benardete who has tried to write a rather complete and critical treatise on "Infinity" suggests that

> "the whole history of mathematics might almost be written around the concept of infinity, the central theme being the various postures adopted toward finitism, both pro and con. Five major phases may be distinguished: (1) the Greeks; (2) the seventeenth and eighteenth centuries with Leibniz and Newton; (3) the nineteenth century under the influence of Gauss; (4) Cantor; and (5) the contemporary crisis in the foundations of mathematics." (5)

As we proceed it might be well for us to keep in mind first a quotation from a letter from Gauss to Schumacher in 1831.

> "...I must protest most vehemently against your use of the infinite as something consumated, as this is never permitted in mathematics. The infinite is but a 'façon de parler'; an abridged form for the statement that limits exist which certain ratios may approach as closely as we desire, while other magnitudes may be permitted to grow beyond all bounds..." (6)

Then we next look at a quotation by Cantor about whom we will say more later.

> "It is traditional to regard the infinite as the indefinitely growing or in the closely related form of a convergent sequence, which it acquired in the seventeenth century. As against this I conceive the infinite in the definite form of something consumated, something capable not only of mathematical formulations, but of definition by number. This conception of the infinite is opposed to traditions which have grown dear to me, and it is much against my own will that I have been forced to accept this view. But many years of scientific speculation and trial point to these conclusions as to a logical necessity and for this reason I am confident that no valid objections could be raised which I would not be in position to meet." (7)

The disturbance that Cantor's remarks created is unequalled in the history of mathematics. It is fortunate for Cantor that "mature reflection had thoroughly steeled him to face the onslaught" for mankind has not the reputation for accepting change graciously. With all his certainty Cantor assured us there was no such thing as the last transfinite and left mathematics split basically into two contending camps presently known as the 'formalists' and the 'intuitionists' (with apology to the logicists). For a more complete discussion of "The Anatomy of the Infinite" and its relationship to the "Two Realities" I recommend to you Tobias Dantzig's excellent book "Number, The Language of Science" from which the above quotations were taken.

Man's first contact with infinity is lost in the unwritten history of the race. Our present understanding of it seems to be traced from the Zeno Paradoxes--one of the earliest records of infinite sequences. Dantzig refers to this as the "first crisis in the concept of infinity". It was almost a century later (3rd century B.C.) that Archimedes formulated the first concept of a limit.

The Zeno Paradoxes and Archimedes exhaustion of area are representative of the thinking of the Greeks toward infinity. The way in which Greek thinking dominated human thought in the years which followed is reflected in the simple fact that we have to wait until the 17th century for Cavaleri in 1635 to give us the first formulation of the infinitesimal. In 1638 Galileo came up with a description of the infinite aggregate, and Pascal gave us mathematical induction in 1654. All of this culminated in 1677 when Newton and Leibniz made the first systematic use of infinite series. Then it was almost 200 years before Gauss, Dedekind and Cantor utilized the now modern definition of an infinite set to present scientific theories for the irrational numbers and introduced the transfinites. David Hilbert said of Cantor's achievement with the transfinites that it is "...the finest product of mathematical genius and one of the supreme achievements of purely intellectual human activity." (8) With this Cantorian explanation of the transfinites we know that there is more than one kind of infinity. We also know that the laws we apply to finite numbers no longer apply when working with infinite cardinalities. In our discussion here most of the time we will be using infinite in the denumerable sense, namely, that which can be placed in a one-to-one correspondence with the natural numbers. In some cases it may not make any difference which infinite we use. In general we consider a set to be infinite if it can be placed in a one-to-one correspondence with a proper subset of itself.

This brings us far too hurriedly to the present crisis in the foundations of mathematics which can perhaps best be characterized by the Russel Paradox and the Theorem of Gödel. The formalists and the intuitionists continue their battle "...and yet bridges stand and men no longer walk in two demensions..."

As Dantzig develops man's growing concept of number he says,

> "And yet this magnificant structure was created by the mathematicians of the last few centuries without much thought as to the foundations on which it rested. Is it not remarkable then, that in spite of all the loose reasoning, all the vague notions an unwarranted generalizations, so few serious errors had been committed? 'Go ahead, faith will follow.' were the encouraging words with which d'Alembert kept reinforcing the courage of the doubters. As though heeding his words, they did forge ahead, guided in their wanderings by a sort of implicit faith on the validity of infinite processes.
>
> Then came the critical period: Abel and Jacobi, Gauss, Cauchy and Weirstrauss, and finally Dedekind and Cantor, subjected the whole structure to a searching analysis, eliminating the vague and ambiguous. And what was the net result of this reconstruction? Well, it condemned the logic of the pioneers, but vindicated their faith." (9)

He concludes by saying,

> "The importance of infinite processes for the practical exigencies of technical life can hardly be overemphasized. Practically all applications

> of arithmetic to geometry, mechanics, physics and even statistics involve these processes directly or indirectly. Indirectly because of the generous use these sciences make of irrationals and transcendentals; directly because the most fundamental concepts used in these sciences could not be defined with any conciseness without these processes. Banish the infinite process, and mathematics pure and applied is reduced to the state in which it was known to the pre-Pythagoreans." (10)

So much for our rather cursory view of infinity for the time being. Now let's consider what we mean when we use the term 'reality'. Its customary use in our everyday language refers to that which we experience through our five senses--that which we can see, hear, touch, smell and taste. Yet rare indeed is the person who has not also had experiences which he attributed to his sixth sense (or call it what you will). In this paper I would like to certainly include this in what we call reality. As we proceed we may want to go somewhat beyond that, however, as we consider such an idea as ultimate reality.

Dantzig asks and answers a question we now want to consider.

> "How real are...infinite processes which endow our arithmetic with this absolute generality, which make it the instrument of our geometrical and mechanical intuition, and through geometry and mechanics permit us to express by number the phenomena of physics and chemistry? Well, if reality be restricted to the immediate experiences of our senses, no thinking man, be he mathematician, philosopher or layman, would attribute reality to the concept." (11)

We are as far beyond attributing reality to the experiences of our five senses today as the mathematics of today is to the mathematics of the Greeks of the Classical era.

As mentioned earlier Dantzig refers to two realities. These he calls "subjective reality" and "objective reality". The subjective reality seems to be what could be described as the aggregate of all the sense-impressions of an individual. As to objective reality, the definition varies with each philosophical school. In general he accepts the definition of Poincaré: "What we call objective reality is, in the last analysis, what is common to many thinking human beings and could be common to all." This definition is broad enough to include ESP, pre-cognition, intuition and the like. It can lead to a sequence of experiences which might have as a bound ultimate reality. With these two definitions in mind let us recall that it has been said that "religion is the mother of the sciences." So let us introduce another dimension of reality which we will refer to as the "spiritual dimension". In the sense that reality is that which is "common to many thinking human beings and could be common to all" certainly the religious idea of a spiritual dimension can be reasonably considered. J.B. Phillips, who makes no claim to being a mathematician, (he is a theologian) says, "We are inclined to think of the physical world,...as somehow real, while the 'spiritual' is regarded as unreal and imaginary." I believe the opposite to be true. As Paul said long ago: "The things which are seen are temporal; but the things which are not seen are eternal." (2 Cor. 4:18) (12) Phillips says, "...the rich variety of transitory beauty (of this world) is no more than a reflection or a foretaste of the real and the permanent." (13)

One of the patterns of mathematical thought that has been most productive has been that of successive approximations. It was the process used by Archimedes to find areas that could be measured in no other way. It was the genius of the development of the calculus. It was the process used to clearly define the irrational numbers and thus place on a relatively secure foundation the real number system. Dantzig says that this process has been at the very heart of the mathematical method.

> "The mathematical method reflected the universe. It had the power to produce an inexhaustible variety of forms. Among these was that cosmic form which some day may embrace the universe in a single sweep. By successive approximations science would eventually attain this cosmic form, for with each successive step it was getting nearer and nearer to it. The very structure of mathematics guaranteed this asymptotic approach, since every successive generalization embraced a larger portion of the universe, without ever surrendering any of the previously acquired territory." (14)

Now we do not want to ignore the question as to whether the universe to which we refer is finite or infinite, nor do we want to ignore the gap theory associated with knowledge of the infinite, but let's pick up on the asymptotic approach pattern and consider how it might help us relate infinity and reality. Before we do we might note in passing that "mathematics and experiment reign...but an all-pervading skepticism has affected their validity. Man's confident belief in the absolute validity of the two methods has been found to be of an anthropomorphic origin; both have been found to rest on articles of faith...this validity...may rest on no firmer foundation than the human longing for certainty and permanence." (15)

In a book entitled "The Rational and the Superrational" Cassius J. Keyser waxes rather eloquent when he says,

> "What knowledge destroys is ignorance but not emotion...No one that has seriously sought to understand knowledge or to know the ultimate nature of understanding; no one that has tried to penetrate the secret recesses of logical implication, to thread the inmost mazes of ideal relationships and to feel in their essence the subtle affinities of thought; no one that has keenly realized the indissoluble interlocking of thought with thought independently of temporal circumstance or human purpose or will; no one that has clearly beheld in the silent light of meditation great cathedrals of doctrine poised in eternal calm above and upon the spiritual basis of a few select ideas; no one that thus has had a vision or even a glimpse of abiding reality under the changeful garment of the world; no such person can fail...to perceive and to feel the supreme religious emotions of reverence and love and awe, so far from depending upon ignorance, are but elevated, amplified and deepened by the mysteries and the wonders more and more disclosed in the brightening light of knowledge." (16)

He goes on to state that his thesis is that "the Rational implies and reveals the Superrational." He says,

> "...as rational knowledge advances, as the light of reason spreads and intensifies, it more and more reveals evidences and intimations that over and above reason's domain, overarching and encompassing it about, there lie regions of reality unto which the rational nature of man indeed

> aspires, approximates and points, as unto its ideal and over-world, but which it can never attain, much less subdue to the ways of common knowledge, or the familiar forms of thought." (17)

What Keyser seems to be saying here as a mathematician seems to be very much like what Phillips is saying as a theologian. True reality is something we will never experience in our time-space predicament. We can only approach it asymptotically. We can only get glimpses of it which can give us successive approximations of what it is really like. "The realm of things perceived has for its border the realm of the conceived." (18)

Time will not permit here that we explore in any detail some of the ways in which we might utilize the concept of infinity to help clarify certain religious or theological concepts. If you would like to read more along this line, you may want to take a look at Keyser's book, "The Rational and the Superrational" where in a chapter entitled "The New Infinite and the Old Theology" he tries to do just this at some length. For our purposes here it will have to be sufficient to mention a couple of areas he explores. One is in the idea of the triune God--three in one. Here he draws on the transfinites. "...we have here three infinite manifolds...no two of which have so much as a single element in common, and yet the three together constitute one manifold...exactly equal in wealth of elements to each of its infinite components." He hastens to add.

> "Have we proved that there is a Trinity composed of three components related to one another and to the Trinity as the dogma asserts? No. We have proved that the conception of such a Trinity...is rigorously thinkable, perfectly possible and rational (in the area of the transfinites)." (19)

His other examples deal with space concepts--dimensionality, hyperspace, and modern concepts of geometry. The role of non-Euclidean geometries in expanding our concept of space is a classic example of our asymptotic approach to reality.

"Mathematics, even in its purest and most abstract estate, is not detached from life...Mathematics is precisely the ideal handling of the problems of life...which...give it its interest and problems, and its order and rationality...what is known in mathematics under the name of limit is everywhere in life under the guise of some idea...The mathematical concept of invariance and that of infinitude...what are they but mathematizations of that which has ever been the chief of life's hope and dreams... the finding of worth that abides, the finding of permanence in the midst of change, and the discovery of the presence, in what seemed to be a finite world, of being that is infinite." (20) These are the inspired words of Cassius J. Keyser who had a vision of reality as it approached the ultimate reality only to reach it at points at infinity.

We close with a poem (perhaps not the best metric verse you will ever hear)--a poem written by Keyser himself.

Beneath the whole a basal zone;
Sense supports not thought alone,
For ways of Reason point above
Towards Perfect beauty, wisdom, love;

High and vast beyond compute,
A realm of Being absolute,
Supernal source of lights that glow
In radiant tremors felt below.

Reason's glory is in her Dream,
Her highest Truth and Worth supreme
Intimate and half reveal
What they are, in what we feel.

Not in the jungles of the mind
Religion's well-spring shall we find.
Not of Darkness is her might
But of the mystery of Light.

Nay, Thrill and Awe with Grace and Love
Eternal flow from Founts above
The vale of Sense and Thought's confine
To make our common life divine.

Illusion all? How are we blind
To deem illusion of the mind
The Holy Light by which we see,
The sheen of Ideality.

The Light and Soul of what we mean,
What is Felt is what is seen,
The hid Intent of thought, unfurled,
The Glory of the Overworld.

"To debate the 'existence' of such a world were a vain dispute. In some sense, whatsoever quickens, lures and sustains, exists. Aspiration is not mocked. Reason's unattainable ideals are the light-giving AEther of Life. Therein is the precious and abiding reality of the Overworld." (21)

In the past several hundred years we have seen the Platonic vision reborn in a more permanent form largely due to an increased understanding of infinity and the realm of the transfinites. Mathematics in its ultimate form as the science of intellectual order--of pattern and structure--has given us new insights into what reality is. In an asymptotic way we seem to be approaching the ultimate in the understanding of reality--the real over-world which binds our time-space perceptions. I hasten to add that many of our sensory and extrasensory experiences may in themselves appear to have no connection with the popular view of mathematics, yet when collected and ordered sequentially, the pattern of a bounded sequence emerges. Thus reality--ultimate reality--to really know what's going on here will only be known when we go beyond our finite predicament into the realm of the infinite.

## Footnotes

1. Felix E. Browder, "The Relevance of Mathematics", The American Mathematical Monthly, Vol. 83, No. 4 (Apr. '76) p. 253. (Quoted from "Library of Living Philosophers".)

2. David Hilbert, "On the Infinite." Cited in "Philosophy of Mathematics, Selected Readings" edited by Paul Benacerraf and Hilary Putnam. Prentice-Hall, Inc. Englewood Cliffs, N.J. p. 136, 1964.

3. Tobias Dantzig, "Number, The Language of Science" 4th Ed. The MacMillan Company, New York, p. 130, 1954.

4. Cassius J. Keyser, "The Rational and the Superrational" Scripta Mathematica, Yeshiva University, New York, pp. 171-72. 1952.

5. Jose A. Benardete, "Infinity." Clarendon Press, Oxford, p. 14. 1964.

6. Dantzig, op. cit., p. 211

7. Ibid., p. 211

8. Hilbert, op. cit., p. 139

9. Dantzig, op. cit., p. 136

10. Ibid., p. 137

11. Ibid., p. 237

12. J.B. Phillips, "For This Day", edited by Denis Duncan, Word Books, Publisher, Waco, Texas. p. 34. 1974.

13. Ibid., p. 36

14. Dantzig, op. cit., p. 329

15. Ibid., p. 320

16. Keyser, op. cit., p. 30

17. Ibid., p. 32

18. Ibid., p. 38

19. Ibid., p. 101

20. Ibid., pp.76-77

21. Ibid., pp. 45-46

# GETTING THEIR INTEREST - INITIATING STUDENTS INTO THE STUDY OF FOUNDATIONAL ISSUES IN MATHEMATICS

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Jack Nicklaus, the famous golfer, was once asked for advice on choosing a vocation. He is reported to have said, "Find out what it is that you would do for nothing, and then find someone who will pay you to do it." That's good advice. I'm not sure I'd teach mathematics for nothing. And even if I would, I'd rather you didn't let my college know. But I do enjoy teaching mathematics; teaching it to those who love it; teaching it to those who are indifferent; and teaching it to those who dare me to try. I now share with you an example of how I have tried to introduce foundational issues in mathematics to the first group of students, assuming for the present that senior math majors fall into this category. I have some hard earned convictions that such issues can be made palatable, yes, even exciting, to the other two groups of students, but that is another story.

I do not suggest that the example I am about to present should have normative force. I share it with you simply as a report of one teacher's experience. Consider it as being one case study, and a rather odd one at that.

My example can best be described by the phrase "Getting their interest," and it is based on two basic assumptions about the nature of education. Actually, the word assumption is too weak; conviction is a better choice. I must make these two convictions explicit or else my example may not make much sense.

My first underlying conviction is that education involves <u>initiation</u>. This is a concept of education emphasized by R. S. Peters. He expresses it as follows:

> To be educated is not to have arrived at a destination; it is to travel with a different view. What is required is not feverish preparation for something that lies ahead, but to work with precision, passion and taste at worthwhile things that lie to hand. These worthwhile things cannot be forced on reluctant minds, neither are they flowers towards which the seeds of mentality develop in the sun of the teacher's smile. They are acquired by contact with those who have already acquired them and who have patience, zeal, and competence enough to initiate others into them.[1]

[1] R.S. Peters, "Education as Initiation," <u>Philosophical Analysis and Education</u>, edited by R. D. Archambault (London: Routledge & Kegan Paul, 1965), p. 110.

As an undergraduate student I thought that college professors were superhuman. They had an inexhaustible supply of knowledge that just spilled over at class time. And my model of the student-teacher relationship was consistent with this warped view. The teacher had the answers, the student didn't. And so an active teacher gave the answers to a passive student, who gave them back to the teacher a few weeks later, and then promptly forgot most of them.

But now that I am a teacher I view this role quite differently. I now view myself as a learner who is somewhat older than the learners sitting in front of me, probably more experienced than them in many academic areas, possibly less experienced in other areas. And I view my most fundamental task as that of initiating the student into new areas of learning, helping him to get started, in the hope that someday he will become independent of me and will then in turn feel the burden for initiating others.

My second underlying conviction is that Christian liberal education must seek <u>integrative understanding</u>. If you look in the catalogs of most Christian liberal arts colleges you will see reference of some kind to the "integration of faith and learning." But what does all that mean? Is it just contentless jargon intended to impress those who read college catalogs? If you ask five different faculty members at such colleges what these slogans mean, you're liable to get five different answers. And there may indeed be room for a plurality of answers. But is it possible that the plurality reflects more the fact that Christian colleges have not adequately struggled with these concepts? I am convinced that the most pressing task facing those who have committed themselves to Christian liberal learning is to seek to articulate more clearly the meaning of integration.

The unorthodoxy of my upcoming example reflects at least a portion of what integration means to me. I view the "integration of knowledge" as the search for interrelationships. This search asks at least the following three questions: How do the results of scholarship in the various academic disciplines illuminate each other? How does the Biblical record illuminate the findings of the various academic disciplines? And the oft neglected third question: How does the accumulated knowledge in the various academic disciplines illuminate the Biblical record?

I trust that you will detect my strong feelings about education as initiation and about the integration of knowledge as I now share with you my attempt at introducing math majors to foundational issues in mathematics.

The vehicle for my initiation rites was a senior math requirement entitled Integrative Seminar. The course requirements were simple. By the end of the semester, each student was to hand in a written response to each of the following innocent sounding questions:

1. Is mathematics true?

2. Are numbers created or discovered?

3. Why should you spend your life doing mathematics?

4. What is the place of science in your personal view of reality?

5. Is the method of the mathematician useful for the Christian apologist?

A piece of cake! There would be no weekly assignments. The written responses to questions could be as long or as short as deemed necessary by the student. We would spend the first ten weeks reading and discussing five books and one journal article that the teacher thought might be pertinent to these questions. The last five weeks would be spent in collateral reading from a bibliography of about ten other works, with voluntary class time devoted to discussion of pertinent sub-questions raised by either the teacher or the students.

If the initiation failed, I knew that five hours the night before the deadline would be an optimistic estimate of the time spent writing responses. But if the initiation worked, I knew the student would soon find out that five years isn't enough time for preparing responses, for as you have probably noticed, my questions touch all three major areas of philosophy. They include questions of ontology and metaphysics (What is there? What is the nature of ultimate reality?) They include questions of epistemology (How can you know what there is? How can you justify knowledge claims made in any area of discourse?) They include questions of axiology and ethics (What is of value? What ought I to do in light of what I believe to be of value?)

You see, I was counting on my students being naive enough philosophically to not notice, at least for the time being, what unreasonable demands I had placed upon them. A piece of cake indeed!

And so we started reading and having discussions. Our first book was Anthony Standen's Science Is A Sacred Cow. This book contains an acknowledgment to the Long Island Railroad for portions of it were written during the interminable delays that seem to plague this train ride in and out of New York City. As you might guess from this acknowledgment, it is written in a very popular style, containing a heavy dose of sarcasm. But beneath the surface some very significant questions are raised. What is science anyway? Does it consist of those endeavors that use the "scientific method?" If so, what is this method? Despite its lack of sophistication, I found this book to be a great "attention getter." It initiated discussion. Some students who had remained almost totally silent for three years were finally expressing themselves.

But the main function of Standen's book was that it set a trap for the math major. It did this by trying to classify the sciences according to the degree of confidence that you can have in their knowledge claims. According to Standen, biology hardly qualified as a science for it didn't use the "scientific method" as he viewed it. The social sciences didn't fare much better; something about lack of empirical base. But physics was another story. Now we're getting someplace. Here we at least have probable opinion. But the best was yet to come. Mathematics is certainly true, or at least Standen said so.

Many students uncritically adopted Standen's views. The fact that the book was assigned by a math teacher increased the temptation to agree once and for all that mathematics is true. But they were in for a surprise. I reminded them of some of the questions they had been exposed to in Philosophy 101. What is the meaning of "truth" anyway? After considerable prompting they seemed to recall at least four theories on the meaning of truth, generally labelled as correspondence, coherence, pragmatic, and existential theories. Things were getting complicated. No easy answers were on the horizon. But they were starting to get interested, and, for the time being, that was my main purpose.

My students were now ready for something heavier on the nature of science. With this purpose in mind I tried a variety of books in the five or six years that I conducted variations of this experiment. My selections for a second book included E. A. Burtt's <u>The Metaphysical Foundations of Modern Science</u>; Carl Hempel's <u>Philosophy of Natural Science</u>; and Karl Popper's <u>Conjectures and Refutations</u>. My choice of Popper reflected some of my own biases toward his falsification approach to testing knowledge claims in science. Again, discussion was good, at times it was heated. But questions seemed to be proliferating.

We now needed to take a much closer look at the fundamental nature of mathematics. Our third book was S. F. Barker's <u>Philosophy of Mathematics</u>, an excellent little book which, unfortunately, is now out of print. This book is not long on mathematical argument, but it is excellent for identifying what the basic philosophical questions are with respect to mathematics, and for setting current debates on foundational issues in mathematics within the context of philosophical debates that have been going on for years. For example, the debate on the nature of number is viewed from the perspective of the classical Greek debate on the status of universals. Questions about the nature of geometries and number systems are related to the classical distinction between a priori and empirical statements, as well as the related distinction between analytic and synthetic statements.

Students found their initial steps into philosophy of mathematics to be difficult ones. By the time we finished Barker's book they were ready for a change of pace. We next Read G. E. Hardy's A Mathematician's Apology. Students were deeply moved by this book. It raised some serious questions in their minds about vocational choices. Should a person do something just because he's good at it? Is there any justification for doing mathematics? They were especially moved by the tragedy near the end of Hardy's life when he felt he had lost his youthful creative powers and decided he then had nothing more to live for. I think my students did some soul searching, hopefully not three years too late.

We next read C. S. Lewis' Miracles. A Preliminary Study, with special emphasis on the questions raised concerning the meaning of probability, and its relationship to the possibility of miracles. In their probability course a year or two earlier my students had learned to calculate numerical probabilities ad infinitum. They were now asking what it all meant. As they had come to expect by now, I pointed out that there were a number of alternative interpretations.

Finally, we read C. R. Verno's "Mathematical Thinking and Christian Theology," taken from the American Scientific Affiliation (ASA) Journal of June, 1968. Verno proposes a presuppositional approach to Christian apologetics, wherein Christian theology seems to emerge from certain fundamental assumptions in a manner analogous to the way a mathematician's theorems are deduced from his axioms. I raised the possible objection that such a view of apologetics insulates one's fundamental assumptions from the possibility of criticism, and also does not take into account the role of experience in making judgments as to the adequacy of any system of thought. Isn't such an approach neglecting crucial empirical questions as to whether Christianity does indeed make sense of experience? By now certain students could hardly contain themselves. Some weren't even anxious to leave when the bell rang.

This ended our assigned reading program. From now on, students were essentially on their own, with as much guidance from me as they sought out. And I would be less than honest if I didn't admit that some now took a five week vacation from my course, or at least four weeks and six days.

But some were initiated. They got excited about foundational issues. They started reading heavy books that weren't even required. Can you imagine that? They started searching for interrelationships between areas of knowledge they had previously compartmentalized. The questions posed had become significant for them. They started struggling with these questions not because I said so, or because the almighty grade depended on it, but because they now needed to find their own answers. I had their interest.

Of course, the big remaining question was "Now that I have their interest, what do I do?" I tried sharing with them the tentative fragmentary results of my own continuing struggle with these same questions. My responses were not much more profound than the ones I had to read. They seemed relieved to know that the teacher didn't have all the answers after all.

One rather obvious shortcoming of my experiment is that it was no more than it claimed to be, an initiation. Too much had to be left out. In particular, not enough time was spent on rigorous mathematical analysis related to foundational issues, analysis of the axiomatic method and the finer points of set theory and mathematical logic. I am now contemplating a two quarter sequence that combines more of the rigors of mathematical analysis with the approach taken in this previous experiment.

An initiation is only a beginning. I barely had time to get some of these students started. As their initiator I would be very content to know that some of them are still struggling with these questions long after Math 461, as I am.

MATH 461 SYLLABUS
INTEGRATIVE SEMINAR
INSTRUCTOR: H. HEIE
THE KING'S COLLEGE
BRIARCLIFF MANOR, NEW YORK

COURSE OBJECTIVES

1. The student should be initiated into an attempt at personal integration of mathematics and the natural sciences in the structure of knowledge as a whole and in a Christian world-view.

2. The student should significantly develop the ability to think logically, critically, and creatively.

TEXTBOOKS

1. A. Standen, Science is a Sacred Cow
E. P. Dutton and Co., New York

2. E. A. Burtt, The Metaphysical Foundations of Modern Science
Doubleday Co., New York, N.Y.

3. S. F. Barker, Philosophy of Mathematics
Prentice - Hall, Inc., Englewood Cliffs, N.J.

4. G. E. Hardy, A Mathematician's Apology
Cambridge University Press, New York

5. C. S. Lewis, Miracles. A Preliminary Study
Macmillan Co., New York

6. C. R. Verno, "Mathematical Thinking and Christian Theology," ASA Journal, June 1968, pp. 37-41

CLASSROOM PROCEDURE

The texts listed above will be read during the semester according to the attached schedule. Class time will be devoted to a discussion of these books. Discussion questions will be distributed one week prior to each discussion period.

GRADING CRITERIA

The student's grade for the course will be based on the quality of his responses to the Integrative Questions listed on the attached page. These responses are to be typewritten and of a length judged to be suitable by the student. These responses are due on the first day of the final examination period. The weekly textbook discussions should help the student in the formulation of his responses. Additional direction is available in the books listed in the Supplementary Readings on the attached page.

INTEGRATIVE QUESTIONS

1. Is mathematics true?

2. Are numbers created or discovered?

3. Why should you spend your life doing mathematics?

4. What is the place of "science" in your personal view of reality?

5. Is the method of the mathematician useful for the Christian apologist?

SUPPLEMENTARY READINGS (On Library Reserve)


1. G. Barbour, Issues in Science and Religion (ch. 6, 8, 9)
   Prentice - Hall, Inc.

2. M. C. Beardsley, Philosophical Thinking, An Introduction (ch. 6, 7)
   Harcourt, Brace, and World

3. J. Bronowski, Science and Human Values
   Harper and Row - Harper Torchbook

4. W. K. Frankena, Ethics (ch. 5)
   Prentice - Hall, Inc.

5. D. Hawkins, The Language of Nature (ch. 1)
   W. H. Freeman and Company

6. C. G. Hempel, Philosophy of Natural Science
   Prentice - Hall, Inc.

7. M. A. Jeeves, The Scientific Enterprise and the Christian Faith
   Inter-Varsity Press

8. W. A. Luijpen, A First Introduction to Existential Phenomenology (ch. 1, 2)
   Duquesne University Press

9. M. Mandelbaum, Philosophic Problems (pp. 87-108, 334-358, 742-756)
   Macmillan Co.

10. W. L. Schaaf, Our Mathematical Heritage
    Macmillan Co.

DISCUSSION SCHEDULE

| | CHAPTERS | DISCUSSION DATES |
|---|---|---|
| Standen | 1-4<br>5-8 | 1/30<br>2/6 |
| Burtt | 1-5<br>6-8 | 2/13<br>2/20 |
| Barker | 1-3<br>4,5 | 2/27<br>3/6 |
| Hardy | -- | 3/13 |
| Lewis | 1-13<br>14-17 | 3/20<br>3/27 |
| Verno | -- | 4/10 |
| Open Discussions Related To Integrative Questions | -- | 4/17<br>4/24<br>5/1 |
| Due Date For Responses To Integrative Questions | -- | 5/8 |

# THE FOUNDATIONS OF MATHEMATICS AND
# THE MATHEMATICS CURRICULUM

Bayard Baylis
The King's College

In teaching the foundations of mathematics within the framework of a Christian college, and particularly that of a Christian liberal arts college, there are two groups of students which must be served. The first consists of the non-mathematics majors--those non-scientifically oriented "general anything" students who, as a catalog might put it, are to receive "an introduction to and an appreciation of the history, foundations, culture and applications of mathematics." The second group consists of the mathematics majors, and the few science majors who have not been frightened away by the calculus. The gulf between these two groups is sufficiently large, I believe, to indicate the use of two different strategies.

It is not difficult to see the incongruity in the hypothetical catalog description. It is impossible to achieve this desired result in one or two freshman mathematics courses. But because of outside parameters, such as the 130 hours required for graduation, it is also impossible to reasonably expect more than a one- or two-semester exposure to mathematics for most students. The problem is compounded still further by the demands of students, and faculty, for relevancy (and what is less relevant than the foundations of mathematics?). And if that were not enough, more and more students are entering college without the basic arithmetic skills needed to function efficiently and effectively in today's world.

How do we teach foundations under these constraints? Do we want to teach foundations to these students? If we answer the second question negatively, there is no need to consider the first. But I am sure most of us can agree with the following statement taken from Christian Liberal Arts Education, Report of the Calvin College Curriculum Study Committee, 1970:

> "Mathematics today is an important part of the intellectual scene, both in its own right and in its use as "the language of science". It has also been important in many stages of Western History--classical Greece, the late Renaissance, the Enlightenment. Its methods and results are interwoven in the intellectual and technological history of the West. It displays a rigor of procedure not to be found in any other discipline. For these reasons, we recommend the continuation of the present requirement."

The problem emerges. We want students to take a mathematics course, to be exposed to mathematics. But there is no concensus as to what should be included in such an experience. Because of this, I feel that many students are getting just that--an experience. I would like to outline three possible solutions to this problem. I am familiar with all three because we have used all three of them at King's. The reason for the changes was (and is) partial success. I do not mean total dissatisfaction but a feeling that, given the "parameters", we could do something better.

When I started at King's, part of the core curricular requirement was a mathematics course entitled, "Nature of Mathematics". The syllabus included logic, set theory, number systems, mathematical method, axiomatic systems, and anything else the instructor could get in. It was offered in small sections, four each semester, with the only differentiation between sections being the instructor. The one exception was a section, offered in the fall, which was designated the majors' section. In this section more depth of material was presented. But even then credit was not given toward the mathematics major.

One of the advantages of this approach is the fact that you know all students are being exposed to the "proper content". (Whether or not you agree with our content is not important. Replace it with your content.) By offering sections with different instructors, the students were able to match their learning styles with the appropriate teaching style. Another advantage to this approach was that the other disciplines and instructors could rely on the fact that the students had been exposed to certain topics, such as logic, which would be useful in composition work.

An obvious administrative disadvantage is the manpower necessary to teach all those sections. To handle eight sections per year you need the equivalent of one full-time instructor. Another disadvantage, which is closely related to this, is the pressure of other introductory level mathematics courses. We found we had to offer a pre-calculus, integrated trigonometry and algebra course to prepare some of our students for the calculus. There was pressure to offer a discrete or finite mathematics course for the business majors. There was pressure to offer more in the way of content courses for elementary education majors. How can a small department offer all these options in addition to the general course? How can the students take another three or six hours of course work? In addition to these pressures, departments at King's were asked to reduce their faculty by the equivalent of one-half person. In light of these parameters, we felt we had to make a change.

The main feature of the alternative that was decided upon is that under the umbrella of one course title, "Nature of Mathematics", with the exception of a small core, content differs from section to section. We chose as our core: logic, set theory, mathematical method and axiomatic systems. We set up four distinct sections which we labeled Majors (M), Humanities and Education (A), Natural Science (N), Social Sciences (S).

In the Majors section, we attempted to go into much more depth in the core material plus adding material on number systems, abstract systems and history. We assumed our majors were prepared for the calculus, which wasn't always a good assumption. In the Natural Science section, in addition to the core material, we attempted to do the elementary function theory. With the core material, which we considered as essential, it was impossible to cover everything that is usually done in an elementary function course. We found we could cover polynomials, rational functions, logarithms, and exponential functions. The trigonometric functions were introduced, but usually only from one perspective--either as circular functions or triangular functions. Analytic geometry and complex numbers became casualties.

In the Social Science section, in addition to the core, we covered some of the usual finite mathematics topics: elementary probability and statistics, linear algebra and linear programming. We also tried to emphasize the logic and set theory more. In the Humanities and Education section, we covered the core material plus material on number systems, history and geometry. In reality, this section was exactly the same course as we had offered under the earlier plan.

The obvious advantage to this program is the ability to offer each student core curricular credit while at the same time offering something relevant, or at least closer to relevancy, but at the same time to expose him to methodology, history and some philosophical implications of mathematics. It offered the administrative advantage of two fewer sections overall per semester, and hence brought our department in line with the cuts that had been made. A disadvantage which some will be quick to point out is the sacrifice of some of the standard content of the standard introductory level course.

Within two years, the outside parameters at our institution changed again with a major revision of the college-wide core curriculum. These changes were made so that we might better achieve our Goals for the Student which include the following:

> "The student should have a reasonable grasp of the major concepts, principles and methods of inquiry in the principle areas within the social sciences, the natural sciences and the humanities. He should be able to see the place of each area in the structure of knowledge as a whole and in the Christian world view, to the extent that he can create meaningful and original relationships among ideas presented in the various areas and can see the relevance of Christianity to each area".

The changes which affected the mathematics department were the dropping of the requirement of a mathematics course and the institution of a new interdisciplinary course entitled, "Fundamental Issues in the Natural Sciences and Mathematics". The new course, offered at the junior-senior level, is wholistic in nature and interdisciplinary and integrative in approach. It deals with the issues of methodology and presuppositions and questions such as, "What is the nature of truth?" Within the framework of questions and problems from the natural sciences, the questions of mathematical truth, a priori or a posteriori, probability and nature of data arise naturally. This approach offers the advantage of placing these questions of the foundations of mathematics within the larger framework of the foundations of science and mathematics and a Christian worldview.

Another advantage is the reduced teaching load in terms of the course, "Nature of Mathematics". We are moving in the direction of "more relevant" service courses, which we hope will attract students. One obvious disadvantage is the possibility of a student not being attracted to any mathematics course. In which case, he can complete a program without a "formal" mathematics course.

Which of these three approaches is best? The first two had some shortcomings. The third has not had enough time yet to prove, or disprove, itself. Other institutions have used these and have been happy with them. For example, Oral Roberts University and Professor Verbal Snook were recently highlighted in Change's Report on Teaching: 3 with a program which had some similarities to the first outlined here. It appears that perhaps there is no ONE answer for all institutions, or even for a given institution over a period of time. But, given the institution and its parameters, there should be a program.

Now that we have settled the matter for the general student, what do we do with the mathematics majors? I am sure that most of us come from institutions and departments which have goals, written or implicit, similar to the following statement from our Goals for the Student:

> "The student should master the principles, techniques, and methods necessary to begin independent work and to make independent judgments in his major field of study. He should have a reasonable understanding of the extent of his major subject, its history, its relation to the other fields, and its place in contemporary culture."

Doesn't that statement commit us to help our students struggle with the foundations of mathematics? Consider another of King's stated goals:

> "The student should develop the ability to think logically, critically, and creatively. This ability entails attacking a new problem, translating it into workable terms, identifying central issues, recognizing underlying assumptions, evaluating evidence, drawing warranted conclusions, and proposing suitable solutions."

To achieve these goals, one cannot rely on one course. The whole curriculum, and each course individually within that curriculum, must be planned with them in mind. One senior seminar is not enough. Without a foundation from all courses, one experience is insufficient. I would like to outline our curriculum and indicate how we feel each point contributes to the ultimate goal. Our majors are required to take 32 hours of mathematics, 12 hours of which is our calculus sequence. This is the foundation of our analysis courses, and is a prerequisite for over half of our majors' courses. In it we hope to acquaint and introduce the student to the process of analytic thinking. This introduction need not be a "formal" one. It can be best accomplished by the general tone of the course, a few well-chosen comments, examples, and problems throughout the course.

Our next majors' requirement is a three-hour axiomatic course chosen from Algebraic Structures, Geometry, Advanced Calculus, or Topology. The emphasis of these courses is on the structure of the subject matter, and in some sense on structure itself. This provides a reference point for the student in later discussions on the nature and foundations of mathematics.

Our majors have 12 hours of mathematics electives which they may choose from Linear Algebra, Probability, Statistics, Computer Science, Introduction to Applied Mathematics, Numerical Analysis, or the other axiomatic courses. The electives should also have input into the foundations question. For example, the concept of modeling and the modeling process are emphasized in Introduction to Applied Mathematics. The computational, manipulative "what works" aspects are emphasized in Numerical Analysis. These are explicit goals of these two particular courses, stated in writing in the course objectives in the syllabi. But I don't think they have to be "preached down the students' throats". Again, it can be by the general tone of the course, the choice of textbook, the choice of supplementary material, whether lectures, assignments, or readings. The student should be able to sense it, if not at the time of the experience, at least upon guided reflection.

You will recall that I said our program consisted of 32 hours. And if you have been counting, you would know I have accounted for 27 hours. Two of those remaining hours are the Senior Seminar, which is run on the outline developed by Harold Heie and described in his paper. What are the other three? The three hours represent a requirement which I feel is somewhat unique as a requirement. We require a junior-senior level course which is entitled, "History and Foundations of Mathematics".

Obviously a one-semester course in the history and foundations of mathematics is a tall order. This course is meant to be an introduction to the historical and foundational framework of mathematics. The development of mathematics did not occur in a vacuum. Technological, political, and sociological developments influenced mathematical developments, and vice versa. An appreciation of mathematics, or a branch of mathematics, is not complete or realized without some grasp of these interrelationships.

This course is not just a "history" course though. It is a "mathematics" course and "significant mathematics" is done throughout the course through the use of appropriate problems and lecture demonstrations. For example, in the discussion of Hamilton, we went through a development of quarterions in class.

For our purposes, I think we have found an excellent text. It is An Introduction to the History of Mathematics, 4th ed., by Howard Eves. I have had to supplement certain "technical aspects." But each chapter concluded with problems which helped the students crystalize the historical concepts and see the difficulties and subtleties that are involved. How many of you have used Fermat's method, or Barrow's method, or Newton's method of fluxions to calculate the slope of the tangent to a curve? That is one of the exercises at the end of the chapter on the development of the calculus.

In addition to problems which were assigned, another major requirement of the course was a paper on a particular mathematician. This paper was to give a biographical sketch, a description of the historical setting of the individual and an analysis of some of the major works of the individual. It was to indicate the influences which affected the individual, and also the effect the individual had on mathematics and society.

The course, although still in the developing stages, has been extremely well-received by our students. One student, who had been wavering between further study in mathematics or chemistry, decided on mathematics. He said that he could identify with some of the thought processes of some of the historical mathematicians. He had not been able to do that in chemistry.

That is our program. What I hope I've indicated to you is that it need not be your program. But you should have a program, a well-thought out program, a working program.

The King's College
Instructor: Dr. Bayard Baylis

Course: History & Foundations of Mathematics (MA 452)
Spring 1977

TEXT: Howard Eves, An Introduction to the History of Mathematics, 4th ed., Holt, Rinehart & Winston (1976)

COURSE OBJECTIVES: Since mathematics is centuries old, a one-semester course in its history and foundations cannot even begin to scratch the surface. This course is meant to be an introduction to the integration of mathematics into its historical framework. The development of mathematics did not occur in a vacuum. Technological, political, and sociological developments influenced mathematical developments, and vice versa. An appreciation of a branch of mathematics is not complete without a grasp of these interrelationships.

Also, since this is a "mathematics" course, a considerable amount of mathematics will be infused throughout the course through problems. By working out problems, the important historical concepts will become more crystalized, the difficulties and subtleties appreciated and understood. They can provide much material for future teachers.

CLASS PROCEDURES AND GRADING CRITERIA: The classes will be lecture-demonstrations, although it is hoped that much discussion will occur. There will be no in-class examinations. For each "period" covered, problems will be assigned, due one week after assignment. There will be a comprehensive final.

Each student is to select a mathematician and in a paper (minimum length 1500 words) describe the historical setting in which the mathematician lived and worked, give a brief biographical sketch, and an analysis of one or two of the major results. It should point out the influences which affected the mathematician, and also what effect the mathematician had on mathematics and society.

The weightings are as follows:

| | |
|---|---|
| Assigned problems | 30% |
| Paper | 40% |
| Comprehensive Final | 30% |

COURSE OUTLINE:

I. Pre-Seventeenth Century
   - A. Numerical Systems
   - B. Babylonian
   - C. Egyptian
   - D. Greek
      - 1. Pythagorean
      - 2. Euclidean
      - 3. Post-Euclidean
   - E. Far Eastern
   - F. European

II. Modern
   A. Seventeenth Century European
   B. Analytic Geometry
   C. Calculus
   D. Eighteenth Century European
   E. Nineteenth Century European

REFERENCES:

1. Blunt, Jones & Bedient, The Historical Roots of Elementary Mathematics, Prentice Hall
2. Boyer, A History of Mathematics, Wiley
3. Eves & Newsome, The Foundations and Fundamental Concepts of Mathematics, Rev. Ed., Holt, Rinehart & Winston
4. Kline, Mathematics in Western Culture, Oxford
5. Kline, Mathematics - A Cultural Approach, Addison Wesley
6. Kramer, The Nature and Growth of Modern Mathematics, Fawcett (paper, 2 vols.), Hawthorne (cloth)
7. Kramer, The Main Stream of Mathematics, Oxford
8. Resnikoff & Wells, Mathematics in Civilization, Holt, Rinehart & Winston
9. Newman, The World of Mathematics, 4 vols., Simon & Schuster
10. Scott, A History of Mathematics, Barnes & Noble
11. Smith, Source Book in Mathematics, 2 vols., Dover
12. Struik, A Source Book in Mathematics, 1200-1800, Harvard
13. Turnbull, The Great Mathematician, Simon & Schuster
14. Weiner, I am a Mathematician; and Ex-prodigy, MIT Press
15. Wheeler, Josiah Willard Gibbs, Yale
16. Wilder, Introduction to Foundations of Mathematics, 2nd Ed., Wiley
17. Wilder, Evolution of Mathematical Concepts, 2nd Ed., Wiley

The King's College
Instructor: Dr. Bayard Baylis

Course: Integrative Seminar (MA 462)
Spring 1977

COURSE OBJECTIVES: To initiate within the student, an attempt at personal integration of mathematics and science as a whole into the structure of knowledge as a whole, and particularly, into a Christian world-view. To deal with some of the significant questions dealing with the foundations of mathematics, science, knowledge and Christianity.

CLASS PROCEDURES: We will meet twice weekly for discussion centered on a pre-assigned reading. Questions that may serve as guides will be provided for each reading, although students are encouraged to develop their own discussion questions.

GRADING CRITERIA: The course grade will be determined by constructive participation in discussions (25%) and by a paper (75%), due Reading Day, in which the following points should be discussed:

1. Is mathematics "true"? What is the nature of "truth"?
2. Is mathematics an art or a science?
3. Is mathematics a priori (predictive) or a posteriori (descriptive)?
4. Is mathematics created or discovered?
5. Compare and contrast the methodology of a mathematician, a scientist, and a Christian apologist.
6. What is the place of mathematics in your view of total reality?
7. State your present plans for the future. Attempt to justify your plans in light of your present system of values.

REQUIRED READINGS:

1. G. H. Hardy, A Mathematician's Apology, Cambridge University Press (1973)
2. C. S. Lewis, Miracles: A Preliminary Study, Macmillan (1976)
3. Alfred Renyi, Dialogues on Mathematics, Holden-Day (1967)
4. William Schaaf, ed., Our Mathematical Heritage, rev. ed., Collier (1963)
5. G. Joseph Wimbish, Readings for Mathematics: A Humanistic Approach, Wadsworth (1972)

RECOMMENDED READINGS:

1. Barbour, Issues in Science and Religion, Prentice-Hall
2. Barker, Philosophy of Mathematics, Prentice-Hall
3. Benacerraf & Putnam, Philosophy of Mathematics: Selected Readings, Prentice-Hall
4. Beardsley, Philosophic Thinking, An Introduction, Harcourt, Brace & World
5. Bell, Development of Mathematics, 2nd ed., McGraw Hill
6. Boyer, History of Mathematics, Wiley
7. Bronowski, Science and Human Values, Harper & Row
8. Frankena, Ethics, Prentice-Hall
9. Hadamard, Psychology of Invention in the Mathematical Field, Dover
10. Hawkins, The Language of Nature, Freeman

11. Howkins, The Challenge of Religious Studies, IVP
12. Kline, ed., Mathematics in the Modern World: Readings from Scientific American, Freeman
13. Kramer, Modern Mathematics: Its Growth and Nature, Hawthorne
14. Luijpen, A First Introduction to Existential Phenomenology, Duguesne Press
15. Mackay, The Clockwork Image, IVP
16. Mendelbaum, Philosophical Problems, Macmillan
17. Newson, Mathematical Discourses: The Heart of Mathematical Sciences, Prentice-Hall
18. Pedoe, The Gentle Art of Mathematics, Macmillan
19. Russel, Introduction to Mathematical Philosophy, George Allen & Urwin, Ltd.
20. Saaty & Weyl, The Spirit and Uses of the Mathematical Sciences, McGraw Hill
21. Stabler, Introduction to Mathematical Thought, Addison-Wesley
22. Standen, Science is a Sacred Cow, Dutton
23. Sullivan, The Limitations of Science, New American Library
24. Weiner, God & Golem, Inc., MIT Press
25. Wilder, Introduction to the Foundations of Mathematics, Wiley
26. Wittgenstein, Remarks on the Foundations of Mathematics, Macmillan

# A CHRISTIAN POINT OF VIEW

A. Wayne Roberts
Macalester College

Does the fact that you are a Christian affect the way that you teach mathematics? People who ask me that question don't usually phrase it in quite that way. They are more likely to ask if there is a "Christian mathematics," and the knowing smile with which they ask the question suggests that they are quite satisfied that mathematics is mathematics, no matter who teaches it. I have set the question more carefully, because my purpose is to argue that a Christian world view can indeed affect the way one teaches mathematics.

Let's say at the outset that a Christian commitment on the part of any teacher should affect the way he or she goes about the task. This is surely part of the reason that we find in the profession so many Christians and so many people whose values have been shaped by Christian influence. But we mean to take our question in a narrow sense, and so we disallow from the beginning answers that might equally apply to any teacher wishing to take Christianity seriously. Our question asks specifically how the teacher's Christian commitment reflects itself in his or her teaching of courses in the standard mathematics curriculum.

When it is realized that I address myself to teachers on the secular as well as the private college campus, there will be some who will feel that if there are places where our private faith will affect the way we teach, these should be identified and then stamped out in the interest of preserving the nonsectarian stance of our public schools. Since I do agree heartily with the principle of not teaching religious faith, whatever the brand, in our public schools, this is a point on which I shall digress for just a moment.

I think it is generally recognized that none of us approach any subject in a completely objective way. While some outsiders might think mathematics to be the one subject where objectivity might be possible, those of us on the inside - and especially those of us who have been involved in discussions such as we have heard in this conference - know differently. The way a mathematician teaches is very much influenced by how he or she views the nature of the subject. One letter to the Notices of the American Mathematical Society decries the lack of concern with applications in the teaching of our discipline, assuring us that virtually all significant advances in our knowledge have been born of attempts to solve practical problems. The next letter is written in the spirit of Jacobi who assured us "that the unique object of science is the honor of the human spirit, and on this basis a question of the theory of numbers is worth as much as a question about the planetary system." In one classroom the student gets the impression that mathematics is a formal discipline built on axioms that have been chosen almost arbitrarily. In the next classroom, the important idea is to get a "feel" for the subject, and we are reminded that de Morgan said of Newton, "so happy in his conjectures as to seem to know more than he could possibly have any means of proving."

The view today, I believe, is not that the college teacher will achieve total objectivity, but that the students will be informed of the "slant" from which the teacher approaches the subject. It is inexcusable, of course, for the teacher to expect all students to adopt the same point of view. That is not the purpose. In fact, it is quite the opposite. The point of view is identified so that the student will be able to evaluate the class within a context. It may also serve the purpose of reminding the teacher that his is just one point of view, and that the students know it.

And so we return to the question of whether or not one's Christian faith affects the way one thinks about mathematics. If it does, then the first obligation of the teacher is to identify Christian faith as one of the influences on his own thinking, hence on the way the class will be taught.

This is where I start, particularly with a freshman class, or with any group that has not been around long enough to hear about my peculiarities. I tell the class something about the community where I grew up, it's orientation toward vocational training, my undergraduate training in a school of engineering, and its influence on my view of mathematics. I tell them of the very conservative church in which I was raised, and of the many times that I was warned not to let Godless professors destroy my faith. I admit that many of my ideas on Christian faith have changed as my education has gone on, but I conclude with what seems to me fair warning. "If it was necessary to warn me that my faith would be challenged by professors who were not Christian, it may be only fair to warn some of you that your nonfaith may be challenged by this professor who is Christian." I tell them that I believe the task of a professor is not to indoctrinate on any topic, whether it be "applied" vs. "pure" mathematics, capitalism vs. socialism, or a Christian vs. non-Christian Weltanschauung. Rather, the task is to raise questions and clarify the options; and it is to the raising of specific questions that we turn.

There is probably no subject in the curriculum which depends more directly on the underlying belief that what seems a sound argument to one person will seem sound to another. Students instinctively believe that in mathematics, there is a right answer, a correct argument. I once signed a drop card for a student who, in answer to my query as to why she was shifting to another course, said "I decided that I prefer subjects where my opinion counts for something." I was persuaded in further questioning that she was not making a veiled criticism of the way the class was run, but was reacting to her perception that there is in mathematics a rigid standard of right and wrong to which everyone will agree.

This matter of what is aptly called common sense, the possibility that we can all agree that a certain answer is right, is an idea I try to emphasize in a variety of ways. I use the paradoxes, pointing out that we recognize them as paradoxes precisely because even though a learned and clever argument seems to lead us to one conclusion, our trust in our own sense of how the matter should turn out is such that the argument to the contrary does not shake our original judgment. I also refer students often to Lewis Carroll's little story, "What the Tortoise Said to Achilles." (This story is conveniently preserved for us, by the way, in Newman's World of Mathematics (3, pp. 2402-2404).) I am also fond of quoting the great thinkers on this subject.

> "...the power of forming a good judgment and of distinguishing the true from the false, which is properly speaking what is called Good Sense or Reason, is by nature equal in all men."
>
> \- Descartes, Discourse on the Method

> "...In every man there is an eye of the soul which, when by other pursuits lost and dimmed, is by these [arithmetic, geometry] purified and re-illuminated; and is more precious far than ten thousand bodily eyes, for by it alone truth is seen."
>
> \- Book VII of Plato's Republic

> "Considering also that of all those who have hitherto sought for truth in the Sciences, it has been the mathematicians alone who have been able to succeed in making any demonstrations, that is to say producing reasons which are evident and certain, I did not doubt . . ."
>
> \- Descartes

> "Only mathematicians are happy men."
>
> \- Novalis

I am not certain what Novalis had in mind with that last quote, but I often suggest that as Descartes put it, only mathematicians have achieved anything like absolute certainty in their work. Naturally I raise the obvious question. To what do you ascribe this capacity for reason that seems unique to the human animal?

Closely associated with this common capacity for reason is the possibility that with some questions, at least, a correct answer is possible. When two (or three) methods emerge as class favorites for a problem, and when these methods give different answers, I sometimes suggest that we decide the matter by voting. When they smile and reject this proposal, I ask them if they think it may be that in some cases at least, there is a way that is right, no matter what popular opinion may hold. When it is allowed that this is surely the case in mathematics, I ask whether this principle (there is, among competing options, an answer or an explanation that is correct) extends to other areas of science. Most of the class usually thinks so, though some demur. The difference of opinion becomes much more pronounced when I ask whether the principle extends to religious questions. I, of course, only ask the question.

The question of whether there is an underlying order to the universe is another question that comes up n a natural way in mathematics. Consider the topic of curve fitting, for example. Plot a number of points in the plane, imagined to be readings taken in some sort of experiment. After posing the problem of trying to model the behavior of the process under examination, draw a wildly oscillating curve that manages (barely) to contain all the plotted points. The reaction of the class is 100% predictable. They think you have taken leave of your senses; they laugh; they think you're putting them on. They are certain that any sensible person would draw as smooth a curve as possible.

Ask them why. Press the point. Suggest that they read Poincare's Science and Hypothesis[4]. Suggest that they read Kuhn's Structure of Scientific Revolutions[2], paying special attention to the idea that one compelling argument for adopting a new paradigm is that it offers a less complicated explanation than did the old one. Again, I keep on hand some favorite quotations.

> "Without the belief that it is possible to grasp the reality with our theoretical constructions, without the belief in the inner harmony of our world, there could be no science."
>
> \- Leopold Infeld and Albert Einstein, <u>Evolution of Physics</u>, Pg. 313

> "Fifty years ago physicists considered, other things being equal, a simple law as more probable than a complicated law. This principle was even invoked in favour of Mariotte's law as against that of Regnault. But this belief is now repudiated; and yet, how many times are we compelled to act as though we still held it! However that may be, what remains of this tendency is the belief in continuity and as we have just seen, were that to disappear, experimental science would become impossible."
>
> \- Henri Poincaré, Science and Hypothesis, Pg. 205-206

It is possible, as Poincaré indicates, to press this point too far. Heisenberg's uncertainty principle, Brownian motion, and other concepts of modern physics demonstrate that our understanding can be aided by a probabalistic approach to science, a point Bronowski has made in The Common Sense of Science [1]. Still, scientists continue for the most part to work as if they believed in an underlying order, students in conscious (and as illustrated above, unconscious) ways believe it, and one certainly has the opportunity of asking why this is so.

This notion of Christian witness is not satisfying to those disposed to the notion that to every question, "we have the answer." To me, on the other hand, one mark of maturity is the capacity to live comfortably with unresolved questions. C.K. Chesterton has said it very well in The Maniac. The mark of an unbalanced mind is that it has neither the good humor nor the good sense to ignore anything. It is without healthy hesitation and healthy complexity.

It is in this sense of learning to live with questions that don't admit to neat answers that I have been helped most by axiomatic thinking. The realization, for example, that one must leave some terms undefined helped me be comfortable with the fact that I could not seem to adequately define my concept of God. The realization that some assumptions are a necessity in any logically developed system helped me accommodate to the fact that none of the so-called proofs of the existence of God seemed really to be proofs as I understood the word. These analogies, moreover, open the door to interesting discussions. I often ask students, at the appropriate time in calculus, what gravity is, whether they believe in it, and why. I ask them how a belief in gravity differs from a belief in God.

The questions and ideas that I have along these lines are expressed in a book I have written called Assumptions and Faith[5]. I clearly cannot expand on all these ideas here, and I admit to increasing reluctance to try, especially in the company of committed Christians. Their first reaction is usually to question whether I have had a personal experience with God, the kind of experience that would have left me knowing that He exists.

I trust I will not be so misunderstood by my present audience. I am telling you that these ideas have helped me. They have helped me be at ease in the company of unsympathetic people when I admit to them that I cannot give what will seem to them an adequate definition of the God I worship, when I admit that I cannot prove to their satisfaction that He exists. And they have led to some very interesting discussions. In this respect, I will tell you that it is in my experience a very helpful thing to have something you have written which can serve as a basis for extended discussions with those who are interested.

I have been in close contact now for about two years with a young lady who is a graduate of the school where I teach. She was coming from a background of no Christian teaching of any kind when we first entered into discussions of the kind I have been outlining. She has, in a series of tentative steps and through a series of personal problems, come to embrace a Christian faith that is (at times) very real to her. I think, in fact, that she is now wondering (like others I mentioned) whether my faith is not too tentative a thing, whether I do not spend too much time just asking questions. Several weeks ago she stopped me after the morning service at our church and asked me, "Dr. Roberts, what do you know for sure?"

I told her that I know that I have made a conscious decision to make Jesus Christ the Lord of my life, and to let His teachings be the principles by which I try to guide my life. I also told her that this decision had given purpose to my life, so that while I was not delivered from the problems that attend the human situation, I knew for sure that my faith had been sufficient to sustain me through all that I have encountered so far. And if I could recall the moment, I would add one more idea. I would tell her that I am sure that this faith has drawn me into the company of people with whom I share a great sense of common mission, a company of people from whom I draw strength and encouragement to do my very best in what we believe to be our common work. This to me is one of the great purposes served by a conference such as the one which we here bring to a close.

## Footnotes

[1] J. Bronowski, The Common Sense of Science, Random House, New York.

[2] T.S. Kuhn, The Structure of Scientific Revolutions, Univ. of Chicago Press, 1962.

[3] J.R. Newman, The World of Mathematics, Simon and Schuster, 1956, New York.

[4] H. Poincaré, Science and Hypothesis, Dover.

[5] A.W. Roberts, Assumptions and Faith, Gibbs, 1974, Broadview, Illinois.